EENADU MODEL EAMCET

ENGINEERING KEY (Revised)

MATHS

1-2; 2-3; 3-2; 4-4; 5-3; 6-1; 7-2; 8-4; 9-4; 10-1; 11-3; 12-1; 13-3; 14-4; 15-1; 16-4; 17-2; 18-1; 19-3; 20-2; 21-4; 22-1; 23-4; 24-2; 25-4; 26-3; 27-2; 28-1; 29-4; 30-3; 31-1; 32-4; 33-2; 34-4; 35-4; 36-2; 37-2; 38-4; 39-3; 40-3; 41-4; 42-4; 43-1; 44-3; 45-2; 46-4; 47-4; 48-2; 49-3; 50-1; 51-3; 52-4; 53-2; 54-2; 55-1; 56-4; 57-3; 58-3; 59-2; 60-2; 61-3; 62-3; 63-3; 64-1; 65-2; 66-2; 67-4; 68-1; 69-1; 70-3; 71-3; 72-1; 73-4; 74-3; 75-2; 76-1; 77-4; 78-3; 79-3; 80-1.

PHYSICS

81-1; 82-2; 83-3; 84-3; 85-4; 86-4; 87-3; 88-2; 89-4; 90-2; 91-3; 92-2; 93-2; 94-1; 95-3; 96-4; 97-3; 98-2; 99-2; 100-4; 101-3; 102-3; 103-1; 104-4; 105-1; 106-3; 107-1; 108-2; 109-4; 110-3; 111-4; 112-3; 113-1; 114-3; 115-2; 116-1; 117-1; 118-1; 119-4; 120-4.

CHEMISTRY

121-1; 122-4; 123-3; 124-4; 125-2; 126-3; 127-4; 128-2; 129-3; 130-4; 131-3; 132-3; 133-4; 134-2; 135-2; 136-4; 137-3; 138-3; 139-2; 140-3; 141-4; 142-4; 143-3; 144-4; 145-1; 146-2; 147-4; 148-3; 149-4; 150-2; 151-4; 152-3; 153-3; 154-4; 155-add score; 156-3; 157-1; 158-3; 159-4; 160-2.

EENADU MODEL EAMCET

ENGINEERING SOLUTIONS (Revised)

MATHEMATICS

1. $1008 = 2^4.3^2.7^1$; Even divisor $\Rightarrow$ Select 2 atleast once and remaining any

$\therefore$ 4.3.2 = 24. Also proper $\Rightarrow$ Excluding $2^4.3^2.7^1$ $\Rightarrow$ Ans = 24 − 1 = 23

2. S = Stop, N = No Stop. Arrange 5 S's and 10 N's in 15 places so that no 2 S's are adjacent $\Rightarrow {}^{11}C_5$ ways.

First arrange 10 N's $\Rightarrow$ 1 way. Then arrange 5 S's in available 11 places $\Rightarrow {}^{11}C_5$ ways.

3. $|z| = |1 + i\tan\alpha| = \sqrt{1^2 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha|$

$= -\sec\alpha$ for $\alpha \in$ III Quadrant

$\therefore |z| = -\sec\alpha$

$|-5iz| = |5|\,|i|\,|z| = (5)(1)(-\sec\alpha) = -5\sec\alpha$

4. $z = \left(\frac{-1}{\sqrt{3}} + \frac{1}{2}i\right)^5 + \left(\frac{-1}{\sqrt{3}} - \frac{1}{2}i\right)^5$

Now replacing i with −i, we get

$\overline{z} = \left(\frac{-1}{\sqrt{3}} - \frac{1}{2}i\right)^5 + \left(\frac{-1}{\sqrt{3}} + \frac{1}{2}i\right)^5 = z$

But $z = \overline{z}$ only when Img $z = 0$

5. $|z| = z + 3 - 2i \Rightarrow \sqrt{a^2 + b^2} = (a + ib) + 3 - 2i$

Real part $\Rightarrow a + 3 = \sqrt{a^2 + b^2}$ and Img part $\Rightarrow b - 2 = 0$

$\therefore b = 2$ and $a + 3 = \sqrt{a^2 + 4} \Rightarrow a^2 + 6a + 9 = a^2 + 4 \Rightarrow a = \frac{-5}{6}$

6. $z = \frac{18\left(-1 - i\sqrt{3}\right)}{1 + 3} = 9\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)$

$iz = 9\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$

$\therefore \cos\alpha = \frac{\sqrt{3}}{2}, \sin\alpha = \frac{-1}{2}$

α is in IV Quadrant $\Rightarrow \alpha = \frac{-\pi}{6}$

7. $\dfrac{6x}{4x-1} - \dfrac{1}{2} < 0 \Rightarrow \dfrac{12x-(4x-1)}{2(4x-1)} < 0 \Rightarrow \dfrac{8x+1}{4x-1} < 0$

So $(8x+1)(4x-1) < 0 \Rightarrow \dfrac{-1}{8} < x < \dfrac{1}{4}$

8. $\dfrac{1}{\sqrt{3x+1}}\left[\left(\dfrac{1+\sqrt{3x+1}}{2}\right)^7 - \left(\dfrac{1-\sqrt{3x+1}}{2}\right)^7\right]$

$= \dfrac{1}{y}\left[\dfrac{(1+y)^7-(1-y)^7}{2^7}\right]$

$= \dfrac{1}{2^7.y}\left[2\left({}^7C_1 y + {}^7C_3 y^3 + {}^7C_5 y^5 + {}^7C_7 y^7\right)\right]$

$= \dfrac{1}{2^6}\left[y^6 + {}^7C_5 y^4 + {}^7C_3 y^2 + 7\right]$

$= \dfrac{1}{2^6}\left[(3x+1)^3 + 21(3x+1)^2 + 35(3x+1) + 7\right]$

It is a 3rd degree polynomial in x.

9. $(1+px)^n = {}^nC_0 + {}^nC_1(px) + {}^nC_2(px)^2 + \ldots\ldots..$

$= 1 + np.x + \dfrac{n(n-1)}{2}\, p^2.x^2 + \ldots\ldots.$

$\therefore$ np = 8 and $\dfrac{n(n-1)\,p^2}{2} = 24$

$\Rightarrow np(np-p) = 48 \Rightarrow 8(8-p) = 48$

$\therefore$ p = 2, n = 4

10. $x^3 + px + q = 0 \Rightarrow \alpha+\beta+\gamma = 0$

So $\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix} = \begin{vmatrix}\alpha+\beta+\gamma & \beta & \gamma\\ \beta+\gamma+\alpha & \gamma & \alpha\\ \gamma+\alpha+\beta & \alpha & \beta\end{vmatrix} = \begin{vmatrix}0 & \beta & \gamma\\ 0 & \gamma & \alpha\\ 0 & \alpha & \beta\end{vmatrix} = 0$ by $C_1 + C_2 + C_3$

11. $A + A^T = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} + \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$= \begin{bmatrix}2\cos\theta & 0\\ 0 & 2\cos\theta\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ I_2$

$\therefore \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

12. In every column, the 3 elements are in A.P. $\Rightarrow$ det = 0

13. $3\sin\theta - 4\sin^3\theta = k;\ 0 < k < 1 \Rightarrow \sin^3\theta = k \in (0, 1)$

$3\theta = \sin^{-1} k,\ \pi - \sin^{-1} k$

Hence $A + B = \frac{1}{3}\sin^{-1} k + \frac{\pi - \sin^{-1} k}{3} = \frac{\pi}{3}$

$\therefore C = \pi - (A + B) = \frac{2\pi}{3}$

14. $\frac{a}{\tan A} + \frac{b}{\tan B} + \frac{c}{\tan C} = \frac{2R\sin A}{\sin A/\cos A} + ...$

$= 2R(\cos A + \cos B + \cos C)$

$= 2R\left[1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right] = 2R\left(1 + \frac{r}{R}\right) = 2(R + r)$

15. Formulas. Both are correct.

16. a, b, c = 6, 8, 10 $\Rightarrow s = \frac{6 + 8 + 10}{2} = 12$

$\Rightarrow \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(12)(6)(4)(2)} = 24$

$\therefore R = \frac{abc}{4\Delta} = \frac{(6)(8)(10)}{(4)(24)} = 5$

17. $\cosh x = \log\left(x + \sqrt{x^2 - 1}\right)$ is given as $\log\left(2 + \sqrt{3}\right) \Rightarrow x = 2$

18. $\text{Tan}\,\theta.\ \text{Tan}\left(120° - \theta\right).\ \text{Tan}\left(120° + \theta\right) = \text{Tan}\,3\theta = \frac{1}{\sqrt{3}} = \text{Tan}\,\frac{\pi}{6}$

$3\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{18}$

19. For any II Quadrant angle, $|\sin x| = \sin x,\ |\cos x| = -\cos x$

$\therefore y = |\cos x| + |\sin x| = -\cos x + \sin x$ when $x \in \left(\frac{\pi}{2}, \pi\right)$

$\therefore \frac{dy}{dx} = \sin x + \cos x$. Now put $x = \frac{2\pi}{3} \Rightarrow \frac{\sqrt{3}}{2} - \frac{1}{2}$

20. $f(x + y) = f(x).\ f(y)$ By taking $y = 0$, we get $f(0) = 1$

$$f'(5) = \lim_{h \to 0} \frac{f(5 + h) - f(5)}{h}$$

$$= \lim_{h \to 0} \frac{f(5).f(h) - f(5)}{h} = f(5). \lim_{h \to 0} \frac{f(h) - f(0)}{h - 0}$$

$$= f(5).f'(c) = 2 \times 3 = 6$$

(OR)

Use formula $f'(x) = f(x).f'(0)$ for the function having $f(x + y) = f(x).\ f(y)$

21. $f(x) = \log_{x^2} (\log x) = \dfrac{\log (\log x)}{\log (x^2)} = \dfrac{\log (\log x)}{2 \log x}$

$$f'(x) = \frac{\left(2 \log x\right)\left(\frac{1}{\log x} \frac{1}{x}\right) - \left(\log \log x\right)\left(2 \frac{1}{x}\right)}{4 (\log x)^2} = \frac{\frac{2}{x} - \frac{2 \log (\log x)}{x}}{4 (\log x)^2}$$

Put $x = e$, $\log x = \log e = 1$ and $\log \log x = \log 1 = 0$

$$f'(e) = \frac{\frac{2}{e} - 0}{4} = \frac{1}{2e}$$

22. $f(x) = \log (\sin x);\ x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$; f is continuous, differentiable

for all $x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ and $\sin \frac{\pi}{6} = \sin \frac{5\pi}{6} \Rightarrow f(a) = f(b)$

Now $f'(c) = 0 \Rightarrow \left(\frac{1}{\sin x} . \cos x\right)_{\text{at } x = c} = 0 \Rightarrow \cot c = 0 \Rightarrow c = \frac{\pi}{2}$

23. $s^2 = at^2 + 2bt + c \Rightarrow 2s. \dfrac{ds}{dt} = 2at + 2b$

$\therefore\ s.v = at + b \Rightarrow v = \dfrac{at + b}{s} \left(\text{is } \dfrac{ds}{dt} = \text{velocity } v\right)$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{s(a) - (at + b).\frac{ds}{dt}}{s^2} = \frac{s.a - (at + b) . \left(\frac{at + b}{s}\right)}{s^2}$$

$$= \frac{a.s^2 - (at + b)^2}{s^3} = \frac{a(at^2 + 2bt + c) - (a^2t^2 + 2abt + b^2)}{s^3} = \frac{ac - b^2}{s^3}$$

24. Solve $y^2 = 4ax$, $ay = 2x^2$

$$\Rightarrow y^2 = 4ax,\ y^2 = \frac{4x^4}{a^2}$$

$$\Rightarrow 4ax = \frac{4x^4}{a^2} \Rightarrow x^4 = a^3.x \Rightarrow x = 0, a$$

Intersection points are (0, 0), (a, 2a)

At(a, 2a): $2y\frac{dy}{dx} = 4a \Rightarrow m_1 = \frac{4a}{2y} = \frac{2a}{2a} = 1$

and $a.\frac{dy}{dx} = 4x \Rightarrow m_2 = \frac{4x}{a} = \frac{4a}{a} = 4$

$\text{Tan}\ \theta = \frac{4 - 1}{1 + 4.1} = \frac{3}{5}$

25. $y^2 = 4x \Rightarrow a = 1$. End of latus rectum $= (a, 2a) = (1, 2)$ has $t = 1$

Normal equation $y + xt = 2at + at^3$

$\Rightarrow y + x = 2 + 1$, is tangent to circle

$\Rightarrow \frac{|3 - 2 - 3|}{\sqrt{1 + 1}} = \text{Radius } r \Rightarrow r^2 = 2$

26. $x^2 - 4x = 3y - 10 \Rightarrow (x - 2)^2 = 3y - 6 = 3(y - 2)$ has $(h, k) = (2, 2)$ and $a = \frac{3}{4}$

Directrix equation is $(y - k) + a = 0 \Rightarrow (y - 2) + \frac{3}{4} = 0 \Rightarrow y = \frac{5}{4}$

27. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and given points $F_1 = (3, 0)$, $F_2 = (-3, 0)$ are nothing but focus of given ellipse.

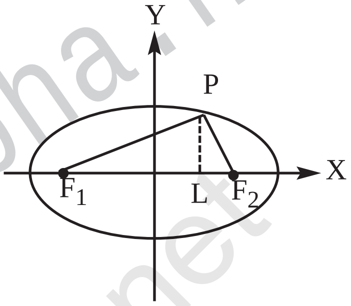
Using $SP + S'P = 2a$, here we get $PF_1 + PF_2 = 2a = 2(5) = 10$

28. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow$ Focus $= (ae, 0) = \left(a.\frac{5}{4}, 0\right)$. It lie on the given focal chord

$2x + 3y = 6 \Rightarrow 2\left(\frac{5a}{4}\right) + 0 = 6 \Rightarrow \frac{5a}{2} = 6 \Rightarrow a = \frac{12}{5}$

Length of transverse axis $= 2a = \frac{24}{5}$

29. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with focus $F_1, F_2 \Rightarrow F_1F_2 = 2ae$



Point on ellipse is $P(a \cos \theta, b \sin \theta)$

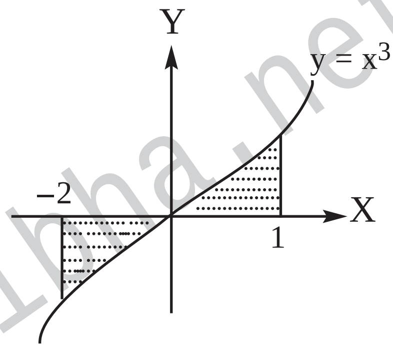
Area of triangle $PF_1F_2 = \frac{1}{2}(F_1F_2)(PL)$

$= \frac{1}{2}(2ae)(b \sin \theta) = (abe)\sin \theta$

Here only θ is variable and maximum value of $\sin \theta$ is 1

Maximum Area $= abe = ab\sqrt{\frac{a^2 - b^2}{a^2}} = b\sqrt{a^2 - b^2}$

30. $y = x^3$ cuts X – axis at $x = 0 \in (-2, 1)$

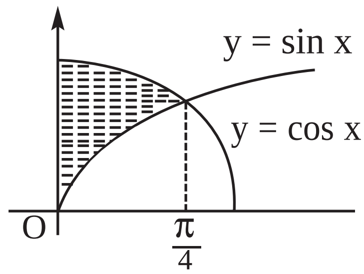


Required Area $= \left|\int_{-2}^{0} x^3\, dx\right| + \left|\int_{0}^{1} x^3\, dx\right|$

$= \left|\left(\frac{x^4}{4}\right)_{-2}^{0}\right| + \left|\left(\frac{x^4}{4}\right)_{0}^{1}\right|$

$= |0 - 4| + \left|\frac{1}{4} - 0\right| = \frac{17}{4}$

31. In $\left[0, \frac{\pi}{2}\right]$, two curves $y = \sin x$,



$y = \cos x$ intersect at $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

Required Area $= \int_{0}^{\pi/4} (\cos x - \sin x)dx = (\sin x + \cos x)_{0}^{\pi/4}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1) = \sqrt{2} - 1$

32. $I = \int_0^{\pi/4} \log\left(\frac{\sin x + \cos x}{\cos x}\right) dx = \int_0^{\pi/4} \log(1 + \text{Tan } x)\, dx$

Hence $I = \int_0^{\pi/4} \log\left[1 + \text{Tan}\left(\frac{\pi}{4} - x\right)\right] dx = \int_0^{\pi/4} \log\left[1 + \frac{1 - \text{Tan } x}{1 + \text{Tan } x}\right] dx$

$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \text{Tan } x}\right) dx = \int_0^{\pi/4} \log 2 - \int_0^{\pi/4} \log(1 + \text{Tan } x)\, dx$

So $I + I = \int_0^{\pi/4} \log 2\, dx = \log 2\left(\frac{\pi}{4} - 0\right)$

$\therefore I = (\log 2)\, \frac{\pi}{8}$

33. $x.\frac{dy}{dx} + 2y = x^2 \Rightarrow \frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$ is linear differential equation

I.F. $= e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$

General Solution is $y(x^2) = \int x.x^2\, dx \Rightarrow yx^2 = \frac{x^4}{4} + c$

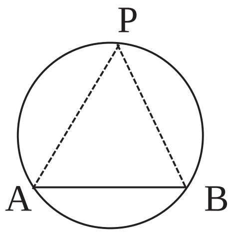
$\therefore y = \frac{x^4 + c}{4x^2}$

34. PQ slope × QR slope = $-1 \Rightarrow \frac{m - 2}{l - 1}.\ \frac{2}{2} = -1$

$\Rightarrow m - 2 = -l + 1 \Rightarrow l + m = 3$ and l, m are natural numbers

Hence $l = 2$, m = 1 (If $l = 1$, m = 2 then P(l, m) = Q(1, 2), not allowed

$\Rightarrow 7l - 3m = 14 - 3 = 11$



35. $\angle APB = 60° \Rightarrow$ P is a point on a circle for which AB is a chord.

Δ APB area is maximum $\Rightarrow$ P is an perpendicular bisector of AB

$\therefore$ PA = PB and $\angle P = 60° \Rightarrow$ Equilateral triangle.

36. $\dfrac{1}{\tan x + \cot x + \sec x + \operatorname{cosec} x} = \dfrac{1}{\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right) + \left(\dfrac{1}{\cos x} + \dfrac{1}{\sin x}\right)}$

$= \dfrac{\sin x.\cos x}{(\sin^2 x + \cos^2 x) + (\sin x + \cos x)} = \dfrac{(\sin x.\cos x)(\sin x + \cos x - 1)}{(1 + \sin x + \cos x)\,[\sin x + \cos x - 1]}$

$= \dfrac{(\sin x.\cos x)(\sin x + \cos x - 1)}{(\sin x + \cos x)^2 - 1} = \dfrac{(\sin x.\cos x)(\sin x + \cos x - 1)}{2 \sin x \cos x}$

$= \dfrac{\sin x + \cos x - 1}{2}$

37. $0 \le x \le \pi$ and $|\cot x| = \cot x + \dfrac{1}{\sin x}$

for $0 \le x \le \dfrac{\pi}{2}$, it become $\cot x = \cot x + \dfrac{1}{\sin x} \Rightarrow \dfrac{1}{\sin x} = 0$, not possible

For $\dfrac{\pi}{2} < x \le \pi$, it become $-\cot x = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow \dfrac{1}{\sin x} = -2 \cot x = \dfrac{-2 \cos x}{\sin x} \Rightarrow -2 \cos x = 1 \Rightarrow \cos x = \dfrac{-1}{2}$

$\therefore \cos x = \dfrac{-1}{2}$ and $\dfrac{\pi}{2} < x \le \pi \Rightarrow x = \dfrac{2\pi}{3} \Rightarrow$ only one solution.

38. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \dfrac{\sin x - (3 \sin x - 4 \sin^3 x)}{-(\cos^2 x - \sin^2 x)} = \dfrac{-2 \sin x + 4 \sin^2 x}{-(\cos 2x)}$

$= \dfrac{-2 \sin x(1 - 2 \sin^2 x)}{-(\cos 2x)} = 2 \sin x$

(OR) Verify by taking $x = 30°$

39. $\mathrm{Cot}^{-1} 21 + \mathrm{Cot}^{-1} 13 + \mathrm{Cot}^{-1} (-8)$

$= \mathrm{Tan}^{-1}\left(\dfrac{1}{21}\right) + \mathrm{Tan}^{-1}\left(\dfrac{1}{13}\right) + \left(\pi - \mathrm{Tan}^{-1} \dfrac{1}{8}\right)$ from $\mathrm{Cot}^{-1}(-x) = \pi - \mathrm{Cot}^{-1}x$

$= \mathrm{Tan}^{-1}\left(\dfrac{\dfrac{1}{21} + \dfrac{1}{13}}{1 - \dfrac{1}{21} \cdot \dfrac{1}{13}}\right) + (\pi - \mathrm{Tan}^{-1} 8)$

$= \mathrm{Tan}^{-1}\left(\frac{34}{273 - 1}\right) + (\pi - \mathrm{Tan}^{-1} 8) = \mathrm{Tan}^{-1} 8 + (\pi - \mathrm{Tan}^{-1} 8) = \pi$

40. $\frac{(x+1)^2}{x^3 + x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1} \Rightarrow (x+1)^2 = A(x^2 + 1) + (Bx + C)x$

Hence 1 = A + B, 2 = C, 1 = A So B = 0

$= \mathrm{Cosec}^{-1} \frac{1}{A} + \mathrm{Cot}^{-1} \frac{1}{B} + \mathrm{Sec}^{-1} C = \mathrm{Sin}^{-1} 1 + \mathrm{Tan}^{-1} 0 + \mathrm{Cos}^{-1} \frac{1}{2}$

$= \frac{\pi}{2} + 0 + \frac{\pi}{3} = \frac{5\pi}{6}$

41. $|\overline{a}| = \sqrt{1+4+4} = 3;\ |\overline{b}| = 5$, angle $(\overline{a}, \overline{b}) = \frac{\pi}{6}$

Area of triangle $= \frac{1}{2}|\overline{a} \times \overline{b}| = \frac{1}{2}\left(3.5.\sin\frac{\pi}{6}\right) = \frac{15}{4}$

42. $|\overline{a}| = \sqrt{1+4+9} = \sqrt{14},\ \overline{a}\,.\,\overline{b} = |\overline{b}|^2$ and $|\overline{a} - \overline{b}| = \sqrt{7}$

$\Rightarrow |\overline{a} - \overline{b}|^2 = 7 \Rightarrow |\overline{a}|^2 + |\overline{b}|^2 - 2\,\overline{a}.\overline{b} = 7$

$\Rightarrow 14 + |\overline{b}|^2 - 2|\overline{b}|^2 = 7 \Rightarrow |\overline{b}|^2 = 7,\ |\overline{b}| = \sqrt{7}$

43. Direction cosines are $\frac{2}{3}, \frac{-a}{3}, \frac{2}{3} \Rightarrow \frac{4}{9} + \frac{a^2}{9} + \frac{4}{9} = 1 \Rightarrow a^2 = 1$

$\therefore$ a = 1 (given a > 0 only) $\therefore$ Direction cosines are $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

So unit vector $\hat{p} = \frac{2}{3}i - \frac{-1}{3}j + \frac{2}{3}k$ and hence

vector of magnitude 3 is $\hat{p} = 2i - j + 2k$

44. $\left[\overline{a}\ \overline{b}\ \overline{c}\right] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$

$= 1[(1+x-y) - x(1-x)] - 0\,[.....]\ -1[x^2 - y]$

$= (1 + x - y - x + x^2) + 0 - x^2 + y = 1$ only

45. Let $\overline{a} = xi + yj + zk$ so $\overline{a} \times i = y(-k) + z(j)$

$(\overline{a} \times i)^2 = y^2 + z^2$ and so on.

$\therefore (\bar{a} \times i)^2 + (\bar{a} \times j)^2 + (\bar{a} \times k)^2 = (y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2)$

$= 2(x^2 + y^2 + z^2)$

$= 2|\bar{a}|^2$

46. $\bar{a}, \bar{b}, \bar{c}$ are coplanar $\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix}$

$= x(2 + 1) - (x - 2)(2 - 1) - 1(-1-1) = 0$

$\Rightarrow 3x - x + 2 + 2 = 0 \Rightarrow 2x + 4 = 0 \Rightarrow x = -2$

47. 2 White + 4 Black + 3 Red = 9 Balls

$n(S) = {}^9C_3 = 84$

$$P(\text{2 same colour and 1 different}) = \frac{{}^2C_2 \cdot {}^7C_1 + {}^4C_2 \cdot {}^5C_1 + {}^3C_2 \cdot {}^6C_1}{{}^9C_3}$$

$$= \frac{1.7 + 6.5 + 3.6}{84} = \frac{55}{84}$$

48. Two dice: $n(S) = 36$

Sum is greater than 5 $\Rightarrow E =$ sum 6, 7, ..., 12

$\therefore \bar{E} =$ sum 2, 3, 4, 5

$\Rightarrow \bar{E} = \{(1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1), (1, 4), (2, 3), (3, 2), (4, 1)\}$

$n(\bar{E}) = 10, n(E) = 26$

$$\Rightarrow P(E) = \frac{26}{36} = \frac{13}{18}$$

49. For A, $P(\bar{A}) : P(A) = 8 : 5 \Rightarrow P(A) = \frac{5}{13}$

For B, $P(\bar{B}) : P(B) = 4 : 3 \Rightarrow P(B) = \frac{3}{7}$

Probability for only one of A, B alive $= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$$= \frac{5}{13} \cdot \frac{4}{7} + \frac{8}{13} \cdot \frac{3}{7} = \frac{44}{91}$$

50. $P(A) = 0.7,\ P(B) = 0.4,\ P(A \cap \overline{B}) = 0.5$

$\therefore P(A \cup \overline{B}) = P(A) + P(\overline{B}) - P(A \cap \overline{B}) = 0.7 + 0.6 - 0.5 = 0.8$

and $P(A \cap \overline{B}) = P(A) - P(A \cap B) \Rightarrow\ 0.5 = 0.7 - P(A \cap B)$

$\therefore P(A \cap B) = 0.2$

So $P\left(\dfrac{B}{A \cup \overline{B}}\right) = \dfrac{P[B \cap (A \cup \overline{B})]}{P(A \cup \overline{B})} = \dfrac{P[(B \cap A) \cup \phi]}{P(A \cup \overline{B})} = \dfrac{0.2}{0.8} = \dfrac{1}{4}$

51. $\Sigma p_i = 1 \Rightarrow 0.2 + 4k = 1 \Rightarrow k = 0.2$

Mean $\mu = \Sigma x_i .p_i = 0 + 1(k) + 2(k) + 3(2k) = 9k = 1.8$

52. Mean $= \dfrac{\Sigma x_i}{n} = \dfrac{6 + 7 + ... + 12}{8} = \dfrac{72}{8} = 9$

Variance $= \dfrac{\Sigma(x_i - \overline{x})^2}{n} = \dfrac{9 + 4 + 1 + 9 + 16 + 25 + 1 + 9}{8} = \dfrac{74}{8}$

$= 9 + \dfrac{2}{8} = 9.25$

53. Mean $= \dfrac{3 + 10 + 10 + 4 + 7 + 10 + 5}{7} = \dfrac{49}{7} = 7$

Mean deviation from $\overline{x} = \dfrac{\Sigma |x_i - \overline{x}|}{n} = \dfrac{4 + 3 + 3 + 3 + 0 + 3 + 2}{7} = \dfrac{18}{7} = 2.57$

54. $11^3 + 12^3 + ... + 20^3 = (1^3 + 2^3 + ... + 20^3) - (1^3 + 2^3 + ... + 10^3)$

$= \dfrac{(20^2).(21^2)}{4} - \dfrac{(10^2)(11^2)}{4} = \dfrac{10^2}{4}\left(2^2.21^2 - 11^2\right) = 25(42 + 11)(42 - 11)$

$= (25)(53)(31)$ is odd, multiple of 5, not multiple of 10.

55. 3 linear equations have no solution when two planes are parallel but not coincide.

Take 2^{nd} & 3^{rd} i.e. $x + 2y + 3z = 10,\ x + 2y + az = b$

parallel $\Rightarrow a = 3$ and not coincide $\Rightarrow b \neq 10$

(OR)

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{bmatrix}$$

It is in Echelon form. No solution.

No solution $\Rightarrow r_1 \neq r_2$

Possible only by a = 3 & b $\neq$ 10

56. $\dfrac{x^2 + 6x - 7}{|x + 4|} < 0$; Denominator $|x + 4|$ is non negative

So $x^2 + 6x - 7$ must be negative $\Rightarrow (x + 7)(x - 1) < 0$

$-7 < x < 1$ and also denominator $\neq 0 \Rightarrow x \neq -4$

$\therefore x \in (-7, 1) - \{-4\}$

57. $x^2 - ax + b^2 = 0 \Rightarrow \alpha + \beta = a; \alpha\beta = b^2$

$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = a^2 - 2b^2$

58. $f(x) = \sqrt{3 + 4x - 4x^2} \Rightarrow 3 + 4x - 4x^2 \geq 0 \Rightarrow 4x^2 - 4x - 3 \leq 0$

$4x^2 - 6x + 2x - 3 = (2x + 1)(2x - 3) \leq 0$

$\Rightarrow \dfrac{-1}{2} \leq x \leq \dfrac{3}{2}$ i.e. Domain $= \left[\dfrac{-1}{2}, \dfrac{3}{2}\right]$

59. $f(x) = x^2 + \dfrac{1}{x^2 + 1} = \left(x^2 + 1 + \dfrac{1}{x^2 + 1}\right) - 1$

Using AM $\geq$ GM we get $x^2 + 1 + \dfrac{1}{x^2 + 1} \geq 2$

$\therefore f(x) \geq 2 - 1 = 1$, Range $= [1, \infty)$

60. $\displaystyle\int \frac{x^3 - 1}{x^3 + x}\,dx$; $\dfrac{x^3 - 1}{x^3 + x} = \dfrac{(x^3 + x) - x - 1}{x^3 + x} = 1 - \dfrac{x + 1}{x(x^2 + 1)}$

Also $\dfrac{x + 1}{x(x^2 + 1)} = \dfrac{A}{x} + \dfrac{Bx + c}{x^2 + 1}$

$\Rightarrow x + 1 \equiv A(x^2 + 1) + (Bx + c)x$

$\Rightarrow A + B = 0, C = 1, A = 1$

$\therefore B = -1$

$$\int \frac{x^3 - 1}{x^3 + x}\,dx = \int 1 - \left(\frac{1}{x} + \frac{-x + 1}{x^2 + 1}\right) dx = \int \left(1 - \frac{1}{x} + \frac{x}{x^2 + 1} - \frac{1}{x^2 + 1}\right) dx$$

$$= x - \log x + \frac{1}{2} \log (x^2 + 1) - \mathrm{Tan}^{-1} x + C$$

61. $\int \frac{\cos 8x + 1}{\tan 2x - \cot 2x} dx = \int \frac{2\cos^2 4x}{\left(\frac{\sin 2x}{\cos 2x} - \frac{\cos 2x}{\sin 2x}\right)} = \int \frac{2.\cos^2 4x.(\sin 2x.\cos 2x)}{\sin^2 2x - \cos^2 2x}$

$= \int \frac{\cos^2 4x (\sin 4x)\, dx}{-\cos 4x} = -\int \sin 4x.\cos 4x\, dx = \frac{-1}{2}\int \sin 8x\, dx$

$= \frac{-1}{2}\left(-\frac{\cos 8x}{8}\right) + C = \frac{1}{16} . \cos 8x + C$

62. $\int_{-1}^{1} x^{27}\cos x + \int_{-1}^{1} e^x .dx.$ Since $x^{27}.\cos x$ is odd function, we get $\int_{-1}^{1} x^{27}\cos x.dx = 0$

So Ans $= \int_{-1}^{1} e^x.dx = \left(e^x\right)_{-1}^{1} = e^1 - e^{-1} = e - \frac{1}{e}$

63. $I = \int_0^{\pi} \frac{x.\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$

$I = \int_0^{\pi} \frac{(\pi - x) .\sin^{2n} (\pi - x)}{\sin^{2n}(\pi - x) + \cos^{2n}(\pi - x)} dx = \int_0^{\pi} \frac{(\pi - x) .\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$

$\therefore I + I = \int_0^{\pi} \frac{\pi.\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$

$\Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx = \frac{\pi}{2}. 2\int_0^{\pi/2} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$

$I = \pi\left(\frac{\pi}{4}\right) = \frac{\pi^2}{4}$

64. $y^2 = 4ax$, a is parameter

$\therefore 2y.\frac{dy}{dx} = 4a$. Put it in above equation

$\therefore y^2 = \left(2y.\frac{dy}{dx}\right)x \quad \Rightarrow y = 2x.\frac{dy}{dx} \quad \Rightarrow \frac{dy}{dx} = \frac{y}{2x}$

65. $\frac{dy}{dx} = \frac{y + x.\tan\left(\frac{y}{x}\right)}{x} = \frac{y}{x} + \tan\frac{y}{x}$ is homogeneous

$\therefore \frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dv}{dx} = v.1 + x.\frac{dv}{dx}$

$\therefore$ D.E. become $v + x.\dfrac{dv}{dx} = v + \tan v$

$\Rightarrow \dfrac{dv}{\tan v} = \dfrac{dx}{x} \quad \Rightarrow \log(\sin v) = \log x + \log c$

$\sin v = cx \quad \Rightarrow \sin\left(\dfrac{y}{x}\right) = cx$

66. $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \dfrac{abc}{2} \Rightarrow 2\,abc \begin{vmatrix} 1 & \frac{x_1}{a} & \frac{y1}{a} \\ 1 & \frac{x_2}{b} & \frac{y_2}{b} \\ 1 & \frac{x_3}{c} & \frac{y3}{c} \end{vmatrix} = \dfrac{abc}{2} \Rightarrow \begin{vmatrix} 1 & \frac{x_1}{a} & \frac{y_1}{a} \\ 1 & \frac{x_2}{b} & \frac{y_2}{b} \\ 1 & \frac{x_3}{c} & \frac{y_3}{c} \end{vmatrix} = \dfrac{1}{4}$

But area of required triangle $= \dfrac{1}{2}\begin{vmatrix} 1 & \frac{x_1}{a} & \frac{y_1}{a} \\ 1 & \frac{x_2}{b} & \frac{y_2}{b} \\ 1 & \frac{x_3}{c} & \frac{y3}{c} \end{vmatrix}$

$$= \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$$

67. Line equation $\dfrac{x}{a} + \dfrac{y}{b} = 1 \Rightarrow A(a, 0), B(0, b)$

Triangle OAB, centroid $= \left(\dfrac{a}{3}, \dfrac{b}{3}\right) = (1, 2) \Rightarrow a = 3, b = 6$

Line is $\dfrac{x}{3} + \dfrac{y}{6} = 1 \Rightarrow 2x + y = 6$

68. $135x^2 - 136xy + 33y^2 = 0$. Its pair of angular bisectors, is

$h(x^2 - y^2) = (a - b)xy \Rightarrow -68(x^2 - y^2) = 102.xy$

$-2(x^2 - y^2) = 3xy \Rightarrow 2x^2 + 3xy - 2y^2 = 0$

$\therefore 2x^2 + 4xy - xy - 2y^2 = 0$

$\Rightarrow (2x - y)(x + 2y) = 0$

Given lines makes equal angles with $2x - y + c = 0$, $x + 2y + d = 0$ for any c, d values.

69. $x - 2y - z + 5 = 0$, $x + y + 3z = 6$ are given planes.

Direction ratios of their intersection line = $\overline{n}_1 \times \overline{n}_2$

$$= \begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 1 & 1 & 3 \end{vmatrix} = i(-6+1) - j(3+1) + k(1+2) = -5i - 4j + 3k$$

Direction ratios of the line are $(-5, -4, 3)$ OR $(5, 4, -3)$

$\therefore$ Required line through (2, 3, 1) is $\frac{x-2}{-5} = \frac{y-3}{-4} = \frac{z-1}{3}$

70. $$\frac{h-0}{2} = \frac{k-0}{-3} = \frac{l-0}{4} = \frac{-(0-0+0-29)}{4+9+16} = 1$$

$\Rightarrow$ $h = 2, k = -3, l = 4$

$\Rightarrow$ Foot from origin = $(2, -3, 4)$

71. A) $$\lim_{x\to\infty} \frac{2x + 7\sin x}{4x + 3\cos x} = \lim_{x\to\infty} \frac{2 + 7\left(\frac{\sin x}{x}\right)}{4 + 3\left(\frac{\cos x}{x}\right)} = \frac{2 + 7(0)}{4 + 3(0)} = \frac{2}{4} = \frac{1}{2}$$

B) $$\lim_{x\to 0} \frac{e^{\tan x} - e^x}{\tan x - x} = \lim_{x\to 0} \frac{e^x\left(e^{\tan x - x} - 1\right)}{(\tan x - x)} = \lim_{x\to 0} e^x . \lim_{y\to 0} \frac{e^y - 1}{y}$$

$$= e^0 \times 1 = 1$$

C) $$\lim_{x\to\infty} \left(\frac{x}{x-1}\right)^x = 1^\infty = e^{\lim_{x\to\infty} x\left[\frac{x}{x-1} - 1\right]} = e^{\lim_{x\to\infty} x\left(\frac{1}{x-1}\right)} = e^1$$

D) $$\lim_{x\to 0} \frac{x}{\sqrt{x+4} - 2} = \lim_{x\to 0} \frac{x\left[\sqrt{x+4} + 2\right]}{(x+4) - 4} = \sqrt{0+4} + 2 = 4$$

72. $$f(x) = \begin{cases} ax + b; & x \le 5 \\ x^2; & x > 5 \end{cases}$$

f is continuous at $x = 5 \Rightarrow 5a + b = 25$ ....(i)

f is differentiable at $x = 5 \Rightarrow a.1 + 0 = 2(5) \Rightarrow a = 10$

$\therefore$ From (i) $50 + b = 25 \Rightarrow b = -25$

$$\frac{a}{b} = \frac{10}{-25} = \frac{-2}{5}$$

73. $x = ct,\ y = \frac{c}{t} \Rightarrow xy = c^2$ By diff $x.\frac{dy}{dx} + y.1 = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x} = \frac{-c/t}{ct} = \frac{-1}{t^2}$ at given $t = 2$. So $\frac{dy}{dx} = \frac{-1}{4}$

74. $f(x) = \log(1 + x) - \frac{2x}{1 + x}$

$\therefore f'(x) = \frac{1}{1 + x} - \frac{(1 + x).2 - 2x.1}{(1 + x)^2} = \frac{(1 + x) - 2}{(1 + x)^2} = \frac{x - 1}{(1 + x)^2}$

$\therefore$ f is Increase when $x - 1 > 0 \Rightarrow x > 1$ and decrease when $x < 1$

$\therefore$ Decrease in (0, 1) and increase in $(1, \infty)$.

75. $f(x) = x^{25}.(1 - x)^{75}$ and we have $(x) + (1 - x) = 1$, constant

Also 25 : 75 = 1 : 3 Maximum when $x = \frac{1}{4}$, $1 - x = \frac{3}{4}$ i.e. $x = \frac{1}{4}$

(OR) $f'(x) = x^{25}.75(1 - x)^{74}(-1) + 25.x^{24}(1 - x)^{75} = 0$

$\Rightarrow 25.x^{24}(1 - x)^{74}[-3x + (1 - x)] = 0$

$\Rightarrow 1 - 4x = 0 \Rightarrow x = \frac{1}{4}$ in (0, 1)

(At x = 0, 1 we get f(x) = 0 which is minimum value)

76. $C_1 = (1, 3)$; $r_1 = r$ and $C_2 = (4, -1)$, $r_2 = \sqrt{16 + 1 - 8} = 3$. So $C_1C_2 = \sqrt{9 + 16} = 5$

Circles intersect $\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow |r - 3| < 5 < r + 3$

Hence $-5 < r - 3 < 5$ and $5 < r + 3$

$-2 < r < 8$ and $r > 2 \Rightarrow 2 < r < 8$

77. $x^2 + y^2 - 6x - 6y + 14 = 0$ has $C_1(3, 3)$ and $r_1 = \sqrt{9 + 9 - 14} = 2$

Required circle has centre say C(h, k). Touches y axis $\Rightarrow$ radius = $|h|$

Touches given circle externally $\Rightarrow C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(h - 3)^2 + (k - 3)^2} = |h| + 2$

$\Rightarrow h^2 - 6h + 9 + k^2 - 6k + 9 = h^2 + 4 + 4|h|$

$\Rightarrow k^2 - 10h - 6k + 14 = 0$

$\therefore$ Locus of centre (h, k) is $y^2 - 10x - 6y + 14 = 0$

78. $|x - 2| + |y - 3| = 4$ represents part of the 4 lines.

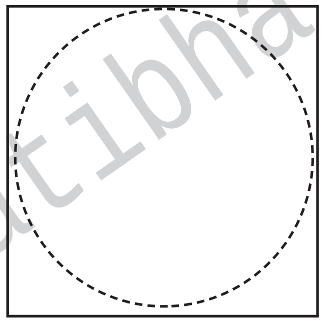
$(x - 2) + (y - 3) = 4, -4$ and $(x - 2) - (y - 3) = 4, -4$

$\Rightarrow x + y = 9, x + y = 1$ and $x - y = 3, x - y = -5$. They forms a square.

The exact middle lines are

$x + y = \frac{9 + 1}{2} = 5$ and

$x - y = \frac{-5 + 3}{2} = -1$



Solving C = (2, 3);

Radius = $\frac{1}{2}\left(\frac{8}{\sqrt{2}}\right) = 2\sqrt{2}$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = 8$

79. $x^2 + y^2 - 2rx - 2hy + h^2 = 0 \Rightarrow (x - r)^2 + (y - h)^2 = r^2$

Tangents from origin are perpendicular $\Rightarrow$ (0, 0) is on director circle

$\Rightarrow$ (0, 0) satisfy $(x - r)^2 + (y - h)^2 = 2r^2$

$\Rightarrow r^2 + h^2 = 2r^2 \Rightarrow r^2 = h^2$

80. $S \equiv x^2 + y^2 - x + 3y - 2 = 0$

$S' \equiv x^2 + y^2 - 3x + y - 2 = 0$ and $S'' \equiv x^2 + y^2 - 4x - 2y - 2 = 0$

Radical Axis of 1st, 2nd circles is $L_1 \equiv S - S' = 0$

$\Rightarrow L_1 \equiv 2x + 2y = 0$

R.A. of 2nd, 3rd circles is $L_2 \equiv x + 3y = 0$

The two Radical axes intersect at origin.

PHYSICS

82. Area = at = $\left(\frac{1}{2}\text{ bh}\right) = \frac{1}{2} \times 6 \times 10 = 30$

$v = u + at = 5 + 30 = 35$

84. $T = m(g - a)$

$\frac{2}{3}mg = m(g - a)$

$a = \frac{g}{3}$

85. $a = 2\ ms^{-2}$

$v_2 = at = 10\ ms^{-1}$

$\overline{v_c} = \frac{m_1\overline{v_1} + m_2\overline{v_2}}{m_1 + m_2} = \frac{0 + 3(10)}{5} = 6$

86. $a = \frac{F - f_k}{m}$

$5 = \frac{F - f_k}{10}$

$18 = \frac{2F - f_k}{10}$

$\frac{5}{18} = \frac{F - f_k}{2F - f_k}$

$18F - 18f_k = 10F - 5f_k$

$8F = 13f_k$ or $F = \frac{13}{8}f_k$

$\therefore 5 = \frac{\frac{13}{8}f_k - f_k}{10} = \frac{5}{80}f_k$

$f_k = 80\text{ N} = \mu_k mg$

$\mu_k = \frac{80}{10 \times 10} = 0.8$

87. $x_c = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = 0$

$\frac{m}{3}x_3 = -\left[\frac{m}{3}(40) + \frac{m}{3}(20)\right]$

$x_3 = -60$

$y_c = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = 0$

$$\frac{m}{3}y_3 = -(m_1y_1 + m_2y_2) = -\left[\frac{m}{3}(0) + \frac{m}{3}(-60)\right]$$

$$\Rightarrow y_3 = +60$$

88. $I = \frac{2}{5} MR^2$

$I' = \frac{2}{5} mr^2$

$\left(\frac{R}{r}\right) = 8^{\frac{1}{3}} = 2$

$\therefore r = \frac{R}{2}, m = \frac{M}{8}$

$\therefore I' = \frac{2}{5}\left(\frac{M}{8}\right)\frac{R^2}{4} = \frac{I}{32}$

89. $\tau = r \times F = \hat{i} \times \hat{j} = \hat{k}$

90. $g_\phi = g - R\omega^2 \cos^2 \phi$

$g_\phi = g_\phi + R\omega^2 \cos^2 \phi$

$= 9.803 + 0.034 \times \frac{1}{2} = 9.820$

91. $F = m\omega^2x^2 = m\frac{4\pi^2}{T^2}x^2$

$F \propto \frac{1}{T^2}$

$\frac{1}{T_1^2} = F_1$

$\frac{1}{T_2^2} = F_2$

$\frac{1}{T^2} = F = F_1 + F_2 = \frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{1}{9} + \frac{1}{16} = \frac{16+9}{144} = \frac{25}{144}$

$T = \frac{12}{5}$

92. $e = \dfrac{T_1 l}{AY}$

$a = \dfrac{120}{60} = 2$

$\therefore T_1 = 10\,a = 20\text{ N}$

$e = \dfrac{20 \times 0.2}{10^{-6} \times 10^{10}} = 4 \times 10^{-4}\text{ m} = 0.4\text{ mm}$

94. $\dfrac{7}{2} = \dfrac{vd_1 + vd_2}{2v} = \dfrac{d_1 + d_2}{2}$

$\therefore d_1 + d_2 = 7$

$\dfrac{12}{7} = \dfrac{m + m}{\dfrac{m}{d_1} + \dfrac{m}{d_2}} = \dfrac{2md_1d_2}{m(d_1 + d_2)} = \dfrac{2d_1d_2}{7}$

$d_1d_2 = 6$

$\therefore d_1 = 1, d_2 = 6$

96. $P_1 l_1 = P_2 l_2$

$(76 + 19)24 = (76 - 19)\,l_2$

$l_2 = \dfrac{95 \times 24}{57} = 40$

97. A, B mixed

$(400)\,s_A\,(6) = (600)\,s_B\,(4)$

$s_A = s_B$

B, C mixed

$600\,s_B\,4 = 800\,s_C\,6$

$s_C = \dfrac{s_B}{2}$

$s_A : s_B : s_C = s_B : s_B : \dfrac{s_B}{2} = 2 : 2 : 1$

98. Slope $\propto \gamma$

Here slope of 2 > slope of 1

γ of 2 > γ of 1

γ of Diatomic = $\frac{7}{5}$ = 1.4

γ of monoatomic = $\frac{5}{3}$ = 1.67

99. $0.5 = 1 - \frac{T_2}{T_1} = 1 - \frac{500}{T_1}$

$\therefore T_1 = 1000$ K

$0.6 = 1 - \frac{T_2}{1000}$

$\therefore \frac{T_2}{1000} = 0.4$, $T_2 = 400$ K

100. $E \propto T^4 \propto \left(\frac{1}{\lambda_m}\right)^4$

$$\frac{E_1}{E_2} = \left[\frac{(\lambda_m)_2}{(\lambda_m)_1}\right]^4 = \left(\frac{0.1}{0.2}\right)^4 = \frac{1}{16}$$

101. $y = A\sin(\omega t - kx)$

$\frac{y}{A} = \sin(\omega t - kx)$

$\sin(\omega t - kx_1) = \frac{y_1}{A} = \frac{1}{2} = \sin\frac{\pi}{6}$

$\phi_1 = \frac{\pi}{6}$

$\sin(\omega t - kx_2) = \frac{y_2}{A} = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$

$\phi_2 = \frac{\pi}{4}$

$\therefore \Delta\phi = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)x$$

$$\frac{\pi}{12} = \left(\frac{2\pi}{\lambda}\right)x$$

$$\therefore\ x = \frac{\lambda}{24}$$

102. $\frac{v}{2l} - \frac{v}{4l} = \frac{v}{4l} = 2$

$$\frac{v}{2\frac{l}{2}} - \frac{v}{8l} = \frac{v}{l} - \frac{v}{8l} = \frac{7v}{8l} = \frac{7}{2}\left(\frac{v}{4l}\right) = 7$$

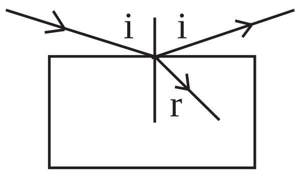
103. $\lambda' = \frac{v + v_s}{n} = \frac{v + \frac{v}{4}}{n} = \frac{5v}{4n} = \frac{5}{4}\lambda = \frac{5}{4}\ (48) = 60$ cm

104. $d = i - r = 26°$

$i + r = 90°$

$2r = 64$

$r = 32°$



105. $i_1 = 0°$

$r_1 = 0°$

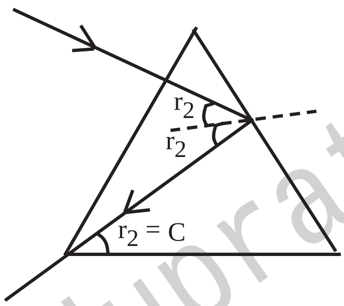
$r_2 = A$

Also $2r_2 = C$

$\sin C = \frac{1}{\mu} = \frac{1}{2}$

$\therefore\ C = 30° = 2r^2$

$r_2 = 15°$



106. $(\mu_1 - \mu_2)t = n\lambda$

$(0.3)t = 5 \times 480 \times 10^{-9}$

$$t = \frac{5 \times 480 \times 10^{-9}}{0.3} = 8000 \times 10^{-9} = 8 \times 10^{-6}\ \text{m}$$

107. $\tan i_p = \mu = \sqrt{3}$

$i_p = 60°$

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$$\mu = \frac{\sin i_p}{\sin r} = \sin r = \frac{\sin i_p}{\mu} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

$r = 30°$

108. $V = \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{r}\right)8$

$$r = \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2} = \sqrt{3}\left(\frac{L}{2}\right)$$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{(Q)(2)}{\sqrt{3}\,L} \times 8$$

109. $\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{2}{1}$

If $V_1 = 6$ kV

$V_2 = 3$ kV

$V_1 + V_2 = 9$ kV

111. $\frac{P}{Q} = \frac{l_1}{l_2}$

$l_1 \propto P = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$

$l_1 \propto \frac{l}{r^2}$

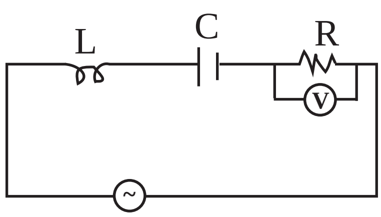
a) $\frac{50}{0.25} = 200$

b) $\frac{100}{1} = 100$

c) $\frac{200}{4} = 50$

d) $\frac{300}{9} = \frac{100}{3}$

113. $i = \frac{100}{10^3} = 100 \times 10^{-3}$ A



$X_C = \frac{1}{C\omega} = \frac{1}{2 \times 10^{-6} \times 200}$

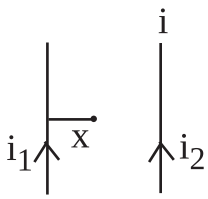
$$= \frac{1}{4 \times 10^{-4}} = \frac{10^4}{4} = 2.5\ \text{k}\Omega$$

$$V_L = V_C = i\,X_C = 2.5 \times 10^3 \times 100 \times 10^{-3}$$

$$= 250\ \text{Volt}$$

114. i) $\frac{i_1}{x} = \frac{i_2}{r - x} \Rightarrow \frac{4}{x} = \frac{8}{6 - x} \Rightarrow 2x = 6 - x$

$$x = 2\ \text{cm}$$



ii) $B = \frac{\mu_0}{2\pi}\left(\frac{i_1}{x} + \frac{i_2}{r - x}\right) = 2 \times 10^{-7}\left(\frac{4}{2 \times 10^{-2}} + \frac{8}{4 \times 10^{-2}}\right)$

$$= 2 \times 10^{-7} \times 4 \times 10^2$$

$$= 8 \times 10^{-5}$$

115. $\lambda = \frac{h}{mv}$

$mv = \frac{nh}{2\pi r}$

$r \propto n^2$

$\therefore\ \lambda = \frac{h}{nh} 2\pi r_3 = 2\pi \frac{n^2}{n} = 2\pi(3x)$

116. $A = \lambda N$

$\frac{A_1}{A_2} = \frac{\lambda_1}{\lambda_2} \frac{N_1}{N_2}$

$\frac{\lambda_1}{\lambda_2} = \frac{\left(T_{1/2}\right)_2}{\left(T_{1/2}\right)_1} = \frac{2}{1}$

$\frac{N_1}{N_0} = \frac{1}{2^2} = \frac{1}{4}$

$\frac{N_2}{N_0} = \frac{1}{2^1} = \frac{1}{2}$

$\frac{N_1}{N_2} = \frac{2}{4} = \frac{1}{2}$

$\therefore\ \frac{A_1}{A_2} = \frac{2}{1} \times \frac{1}{2} = \frac{1}{1}$

118. LSB frequency = $f_c - f_m = 10^5 - 3 \times 10^3$

$= 10^3(100 - 3) = 97$ kHz

USB frequency = $f_c + f_m = 10^5 + 3 \times 10^3$

$= 10^3(100 + 3) = 103$ kHz

120. $i_L = \dfrac{V_L}{R_L} = \dfrac{50\ \text{V}}{10\ \text{k}\Omega} = 5$ mA

(i) i_z Minimum

$V_s = 30$ V, $i_s = \dfrac{V_s}{R_s} = \dfrac{30\ \text{V}}{5\ \text{k}\Omega} = 6$ mA

$i_z = i_s - i_L = 1$ mA

(ii) i_z Maximum

$V_s = 70$, $i_s = \dfrac{70\ \text{V}}{5\ \text{k}\Omega} = 14$ mA

$i_z = i_s - i_L = 14 - 5 = 9$ mA

CHEMISTRY

121. $E = E_0 + E_k$; work function $E_0 \propto \frac{1}{\lambda_0}$; $E_0 = \frac{hc}{\lambda_0}$; $\lambda_0 = \frac{hc}{E_0}$

E_0 is the least for k and hence λ_0 is the highest

$\lambda_0(k) = \frac{1240 \text{ eVnm}}{2.2 \text{ eV}}$ = 563 nm can emit electron from 'k' only

E_0 is the highest for Cu and hence λ_0 is the least

$\lambda_0(Cu) = \frac{1240 \text{ eVnm}}{4.8 \text{ eV}}$ = 258 nm, can emit electron any of the above metals.

122. Half of the total electrons in an atom have identical spin if has no unpaired electrons.

$_{56}Ba$ has 28 electrons of identical spin.

125. C_2; 12 e^-; $\sigma 1s^2 \sigma^* 1s^2 \ \sigma 2s^2 \sigma^* 2s^2 \left[\pi 2p_x^2 \ \pi 2p_y^2\right] \sigma 2p_z^0$ ....

126. $2\,KMnO_4 + \underline{5\,H_2O_2} + 3\,H_2SO_4 \longrightarrow K_2SO_4 + 2\,MnSO_4 + 8\,H_2O + \underline{5\,O_2}$

acid medium 1 mole H_2O_2 gives 1 mole O_2

$2\,KMnO_4 + \underline{3\,H_2O_2} \longrightarrow 2\,MnO_2 + 2\,KOH + 2\,H_2O + \underline{3O_2}$

In basic medium 1 mole H_2O_2 gives 1 mole O_2.

129.

C – C bond lengths	Benzene	Graphite	Fullerene	Diamond
	139 pm	141.5 pm	143.5 pm	154 pm

130. Equatorial P – C*l* bond length   axial P – C*l* bond length

135. μ = 5.92 BM; There are 5 unpaired electrons in the species

Mn^{2+} : $[Ar]\ \underline{3d^5}$

141. $\underset{x\,mL}{CO} + \underset{\frac{x}{2}\,mL}{\frac{1}{2} O_2} \longrightarrow \underset{x\,mL}{CO_2}$; $\underset{(10-x)mL}{CO_2} \xrightarrow{\text{no reaction with } O_2} \underset{(10-x)\,mL}{CO_2}$

$\frac{x}{2}$ = 2.4 mL; x = 4.8 mL

V_{CO} = 4.8 mL; V_{CO_2} = 10 – 4.8 = 5.2 mL; % $CO_2 = \frac{5.2 \times 100}{10}$ = 52

142. In ClO_4^-, the oxidation state of Cl is +7 which is the highest and hence it cannot increase further. ClO_4^- cannot undergo disproportionation.

143. $E_k = \frac{3}{2} nRT = \frac{3}{2} \frac{\omega RT}{M}$

$\therefore E_k \propto \frac{1}{M}$

E_k is the least for gas of the highest molecular weight

$CO_2 = 44$; $N_2O = 44$; $Cl_2O = 87$; $SO_3 = 80$

145. Density $= \frac{Z.\,A}{a^3.N_0}$; $\frac{A}{N_0}$ = Weight of one atom in g $= \frac{\text{weight of crystal}}{\text{number of atoms}}$

$d = \frac{z}{a^3} . \frac{W_{crystal}}{\text{no. of atoms}}$; number of atoms $= \frac{4 \times 54}{\left(300 \times 10^{-10}\right)^3 \times 8} = 10^{24}$

146. Acetic acid in benzene

$i = \frac{\Delta T_{f_{obs}}}{\Delta T_{f_{cal}}} = 1 - \alpha \left(\frac{n-1}{n}\right)$

$\therefore \alpha = \left(1 - \frac{\Delta T_{f_{obs}}}{\Delta T_{f_{cal}}}\right). \frac{n}{n-1}$

$\alpha = \left(1 - \frac{\Delta T_{f_{obs}}}{K_f \times \text{molality}}\right)\left(\frac{n}{n-1}\right)$

$= \left(1 - \frac{0.55}{5 \times 0.2}\right) \frac{2}{2-1} = 0.9$

Given % association of acetic acid (100 α) is same as % ionisation of $MgSO_4$ (100 α)

$MgSO_4$ in water

$i = 1 + (n-1)\ \alpha = \frac{\Delta T_{b_{obs}}}{\Delta T_{b_{cal}}} = \frac{\Delta T_{b_{obs}}}{K_b \times \text{molality}}$

$\Delta T_{b_{obs}} = [1 + (2-1)\,0.9] \times 0.5 \times 2 = 1.9$ K

$T_b^s - T_b^o = \Delta T_{b_{obs}} = 1.9$ K

$T_b^s = T_b^o + 1.9 = 373.15 + 1.9 = 375.05$ K

147. $N_2O_4(g) \rightleftharpoons 2\,NO_2\,(g)$ $\quad p_{NO_2} = 1\text{ atm} \times \frac{100}{150} = \frac{2}{3}\text{ atm}$

i 100 0

r −50 + 2 × 50

equating $\frac{50}{}$ $\frac{100}{}$ $\quad p_{N_2O_4} = 1\text{ atm} \times \frac{50}{150} = \frac{1}{3}\text{ atm}$

$$K_p = \frac{p^2_{NO_2}}{p_{N_2O_4}} = \frac{\left(\frac{2}{3}\text{ atm}\right)^2}{\frac{1}{3}\text{ atm}} = \frac{4}{3}\text{ atm}$$

$$\Delta G = -RT\,ln\,K_p$$

$$= -2.3 \times 2 \times 280 \times 0.125$$

$$= -161\text{ cal mol}^{-1}$$

$$\Delta G = -2.303 \times 2\text{ cal} \times 280\text{ K} \times log\ \frac{4}{3}$$

148. The position of equilibrium does not change by the addition of inert gas like argon if

(1) $\Delta V = 0$ i.e. volume of the system is constant

(or)

(2) $\Delta n_g = 0$ i.e. there is no change in the number of molecules in the gas phase from reactants to products.

149. Given $\frac{pH}{pOH} = \frac{2}{5}$. at 25°C pH + pOH = 14

Hence pH and pOH of the aqueous solutions are 4 and 10 respectively.

$[H^+] = 10^{-pH} = 10^{-4}$ g ion L^{-1}

150. [A] × [B] = Rate

From expt. 1 & 2 $(1)^x \times (2)^y = \frac{19.2 \times 10^{-2}}{9.6 \times 10^{-2}} = 2 \therefore 2^y = 2^1; y = 1$

1 & 3 $(2)^x \times (1)^1 = \frac{38.4 \times 10^{-2}}{9.6 \times 10^{-2}} = 4 \therefore 2^x = 2^2; x = 2$

1 & 4 $(2)^2 \times (2)^1 = \frac{76.8 \times 10^{-2}}{9.6 \times 10^{-2}} = 8 = 2^3$

[A] = 2 × 0.3 = 0.6; [B] = 2 × 0.3 = 0.6

151. $nFE^o = RT\, ln\, K_c; \; E^o = \frac{2.303\, RT}{nF} log\, K_c = \frac{0.06}{n} log\, K_c$

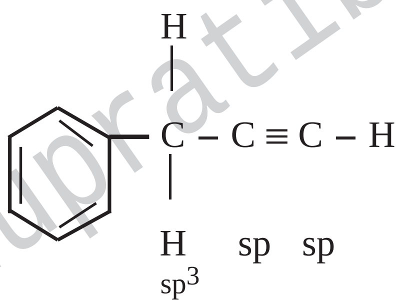
$E^o = \frac{0.06}{2} log\, 10^{16} = \frac{0.06}{2} \times 16 = 0.48\ V$

152. $\frac{W_{Cu}}{W_{Al}} = \frac{E_{Cu}}{E_{Al}} = \frac{A_{Cu}.Z_{Al}}{A_{Al}.Z_{Cu}}; \; W_{Cu} = \frac{W_{Al}}{A_{Al}} . \frac{A_{Cu}.Z_{Al}}{Z_{Cu}}$

$W_{Cu} = n_{Al} . \frac{A_{Cu}.Z_{Al}}{Z_{Cu}} = \frac{10^{-3} \times 64 \times 3}{2} = 96 \times 10^{-3} g$

$= 96\ mg$

154. 3 – Phenyl propyne



155. **Add score**

158. Williamson synthesis reaction is to prepare ethers but not aldehydes.