

BOARD OF INTERMEDIATE EDUCATION
SENIOR INTER MATHEMATICS PAPER - II (B)
MODEL PAPER (ENGLISH VERSION)

TIME: 3 HOURS

MAX.MARKS: 75

Note: This question paper consists of three sections A, B and C.

SECTION – A

I. i) Very Short Answer Type questions.

ii) Answer ALL questions.

ii) Each question carries TWO marks.

10 × 2 = 20

1. If the length of the tangent drawn from the point (5, 4) to the circle $x^2 + y^2 + 2ky = 0$ is 1, find the value of k.
2. Find the length of the chord of the circle $x^2 + y^2 = a^2$ on the line $x \cos \alpha + y \sin \alpha = p$.
3. If $x^2 + y^2 + 4x + 8 = 0$ and $x^2 + y^2 - 16y + k = 0$ represent two orthogonal circles, then find k.
4. If one end of the focal chord of the parabola $y^2 = 8x$ is $\left(\frac{1}{2}, 2\right)$ find the other end.
5. If $3x - 4y + k = 0$ is the tangent to the hyperbola $x^2 - 4y^2 = 5$, then find k.

6. Evaluate: $\int \frac{dx}{\cos hx + \sin hx}$

7. Evaluate: $\int \sec^4 x \, dx$.

8. Evaluate: $\int_0^a \frac{dx}{x^2 + a^2}$

9. Find the area bounded by the curve $|x| + |y| = 1$.

10. Find the degree and order of $1 + \left(\frac{d^2y}{dx^2}\right)^2 = \left[2 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

SECTION – B

II. i) Short Answer Type questions.

ii) Answer any FIVE questions.

iii) Each question carries FOUR marks.

5 × 4 = 20

11. Show that the line $x + y + 1 = 0$ touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$. Also find the point of contact.
12. If the line $2x + 3y = 1$ cuts the circle $x^2 + y^2 = 4$ in A and B, find the equation of the circle with AB as diameter.

13. Show that the equation of the chord joining the points α and β on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{x}{a} \cos \left(\frac{\alpha + \beta}{2} \right) + \frac{y}{b} \sin \left(\frac{\alpha + \beta}{2} \right) = \cos \left(\frac{\alpha - \beta}{2} \right)$$

14. Find the equation of the tangent to the ellipse $9x^2 + 16y^2 = 144$ and which makes equal intercepts on the coordinate axes.

15. Show that the locus of the feet of the perpendiculars drawn from foci to any tangent of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ is the auxilliary circle of the hyperbola.}$$

16. Evaluate: $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{6n} \right]$

17. Obtain the differential equation which corresponds to each of the following family of curves:

- i) The circles which touch the Y – axis at the origin.
ii) The parabolas having their foci at the origin and axis along the X –axis.

SECTION – C

III. i) Long Answer Type questions.

ii) Answer any FIVE questions.

iii) Each question carries SEVEN marks.

5 × 7 = 35

18. Find the locus of mid points of the chords of contact of $x^2 + y^2 = a^2$ from the points lying on the line $lx + my + n = 0$.

19. Find the equations of all common tangents to the circles $x^2 + y^2 = 9$ and $x^2 + y^2 - 16x + 2y + 49 = 0$.

20. Show that the equations of common tangents to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$ are $y = \pm (x + 2a)$.

21. Evaluate: $\int \frac{1}{1+x^4} dx$

22. Evaluate: $\int (6x+5)\sqrt{6+x-2x^2} dx$

23. Evaluate: $\int_0^{\pi} \frac{x \sin^3 x}{1 + \cos^2 x} dx$

24. Solve: $(2x + 4y + 3) \frac{dy}{dx} = (x + 2y + 1)$.