JEE MAINS MODEL PAPER - 2017

No. of Questions: 90 **Maximum Marks: 360** **Time: 3 hours**

PHYSICS

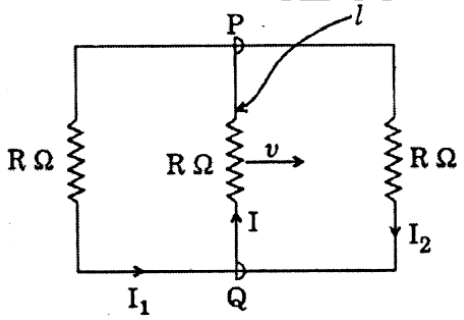
1. A rectangular loop has a sliding connector PQ of length 'l' and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper.

1) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$

2) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

3) $I_1 = I_2 = I = \frac{Blv}{R}$

4) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{2Blv}{3R}$



2. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) 2

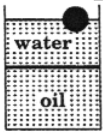
3. Two balls A and B are thrown vertically upwards from the same location on the surface of the earth with velocities $2\sqrt{\frac{gR}{3}}$ and $\sqrt{\frac{2gR}{3}}$ respectively, where R is the radius of the earth and g is the acceleration due to gravity on the surface of the earth. The ratio of the maximum height attained by A to that attained by B is

1) 2 2) 4 3) 8 4) $4\sqrt{2}$

4. A long metal rod of length 'l' and relative density 'σ' is held vertically with its lower end just touching the surface of water. The speed of the rod when it just sinks in water is given by

1) $\sqrt{2gl}$ 2) $\sqrt{2gl\sigma}$ 3) $\sqrt{2gl\left(1-\frac{1}{2\sigma}\right)}$ 4) $\sqrt{2gl(2\sigma-1)}$

5. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

1) 

2) 

3) 

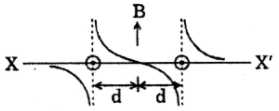
4) 

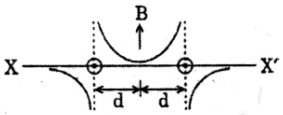
6. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

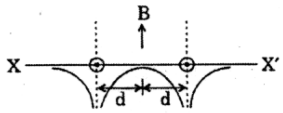
1) $y = x^2 + \text{constant}$ 2) $y^2 = x + \text{constant}$

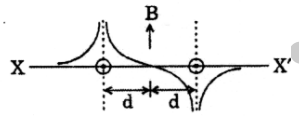
3) $xy = \text{constant}$ 4) $y^2 = x^2 + \text{constant}$

7. Two long parallel wires are at a distance 2d apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by

1) 

2) 

3) 

4) 

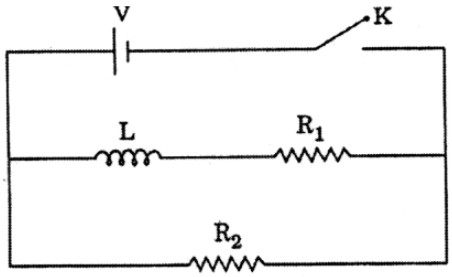
8. In the circuit shown below, the key K is closed at t = 0. The current through the battery is

1) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

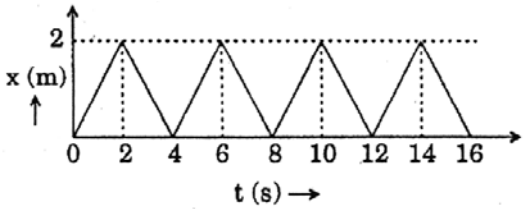
2) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

3) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞

4) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞



9. The figure shows the position – time (x – t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



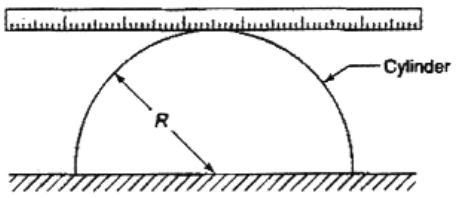
1) 0.4 Ns 2) 0.8 Ns 3) 1.6 Ns 4) 0.2 Ns

10. A nucleus of mass M+ Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

1) $E_2 = 2E_1$ 2) $E_1 > E_2$ 3) $E_2 > E_1$ 4) $E_1 = 2E_2$

11. A uniform metre scale of length 1 m is balanced on a fixed semi-circular cylinder of radius 30 cm as shown in fig. One end of the scale is slightly depressed and released. The time period (in seconds) of the resulting simple harmonic motion is

(Take g = 10 ms^2)

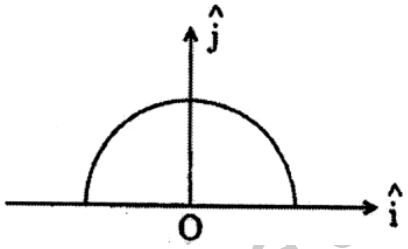


1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

12. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α – particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

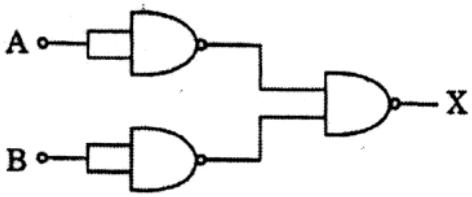
1) $\frac{A - Z - 8}{Z - 4}$ 2) $\frac{A - Z - 4}{Z - 8}$ 3) $\frac{A - Z - 12}{Z - 4}$ 4) $\frac{A - Z - 4}{Z - 2}$

13. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



1) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ 2) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ 3) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ 4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

14. The combination of gates shown below yields



1) OR gate 2) NOT gate

3) XOR gate 4) NAND gate

15. A diatomic ideal gas is used in a Car engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to 32V the efficiency of the engine is

1) 0.5 2) 0.75 3) 0.99 4) 0.25

16. If a source of power 4 kW produces 10^{20} photons/second, the radiation belong to a part of the spectrum called

1) X – rays 2) ultraviolet rays

3) microwaves 4) γ – rays

17. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

1) 5, 1, 2 2) 5, 1, 5 3) 5, 5, 2 4) 4, 4, 2

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

1) 305 W 2) 210 W

3) Zero W 4) 242 W

19. In a transistor circuit, the collector current is 50 mA and the base current is 1 mA. The current gain α is

1) $\frac{51}{50}$ 2) $\frac{49}{50}$ 3) $\frac{50}{51}$ 4) $\frac{49}{51}$

20. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$. D is

1) $\frac{b^2}{2a}$ 2) $\frac{b^2}{12a}$ 3) $\frac{b^2}{4a}$ 4) $\frac{b^2}{6a}$

21. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 gcm^{-3}, the angle remains the same. If density of the material of the sphere is 0.16 gcm^{-3}, the dielectric constant of the liquid is

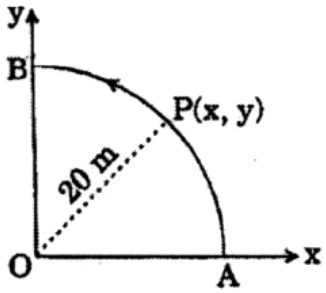
1) 4 2) 3 3) 2 4) 1

22. In the visible region, the dispersive powers and the mean angular deviations for crown and flint glass prisms are ω and ω' and d and d' respectively. When the two prism are combined, the condition of zero dispersion by the combination is

1) $\sqrt{\omega d} + \sqrt{\omega}\ 'd' = 0$ 2) $\omega' d + \omega d' = 0$

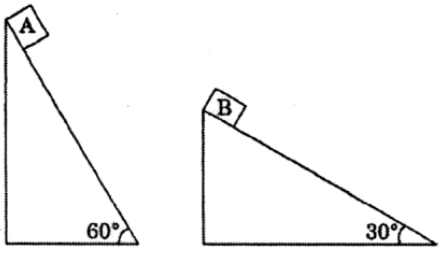
3) $\omega d + \omega' d' = 0$ 4) $(\omega d)^2 + (\omega' d')^2 = 0$

23. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 s is nearly



1) 13 m/s^2 2) 12 m/s^2 3) 7.2 m/s^2 4) 14 m/s^2

24. Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



1) 4.9 ms^{-2} in horizontal direction 2) 9.8 ms^{-2} in vertical direction

3) zero 4) 4.9 ms^{-2} in vertical direction

25. For a particle in uniform circular motion the acceleration a at a point P(R, θ) on the circle of radius R is (here θ is measured from the X – axis)

1) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

2) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

3) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

4) $\frac{v^2}{R}\,\hat{i} + \frac{v^2}{R}\,\hat{j}$

26. A motor car is fitted with a convex driving mirror of focal length 20 cm. A second motor car 2.8 m behind the first car is overtaking at a relative speed of 15 ms^{-1}. The speed of the image of the second car as seen in the mirror of the first is

1) $\frac{1}{10}$ ms^{-1} 2) $\frac{1}{15}$ ms^{-1} 3) 10 ms^{-1} 4) 15 ms^{-1}

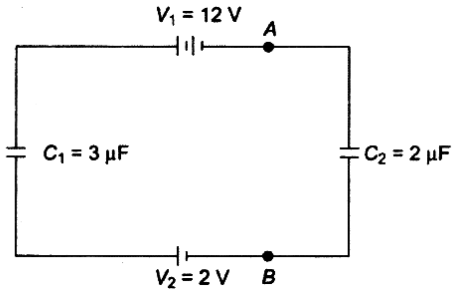
27. The potential difference between points A and B in the circuit shown in fig. is

1) 6 V

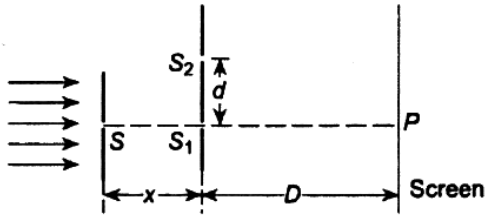
2) 2 V

3) 10 V

4) 14 V



28. Monochromatic light of wavelength λ emerging from slit S illuminates slits S_1 and S_2 which are placed with respect to S as shown in fig. The distances x and D are large compared to the separation d between the slits. If x = D/2, the minimum value of d so that there is a dark fringe at the centre P of the screen is



1) $\sqrt{\frac{\lambda D}{3}}$ 2) $\sqrt{\frac{2\lambda D}{3}}$ 3) $\sqrt{\lambda D}$ 4) $2\sqrt{\frac{\lambda D}{3}}$

29. On a particular day, the maximum frequency reflected from the ionosphere is 8 MHz. On another day it was found to increase to 9 MHz. The ratio of the maximum electron densities of the ionosphere on the two days is

1) $\sqrt{\frac{8}{9}}$ 2) $\frac{8}{9}$ 3) $\left(\frac{8}{9}\right)^{3/2}$ 4) $\left(\frac{8}{9}\right)^{2}$

30. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$y = 0.02(m)\sin\left[2\pi\left(\frac{1}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$ The tension in the string is

1) 4.0 N 2) 12.5 N 3) 0.5 N 4) 6.25 N

CHEMISTRY

31. The standard enthalpy of formation of NH_3 is –46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is –436 kJ mol^{-1} and that of N_2 is –712 kJ mol^{-1}, the average bond enthalpy of N–H bond in NH_3 is

1) –964 kJ mol^{-1} 2) +352 kJ mol^{-1} 3) +1056 kJ mol^{-1} 4) –1102 kJ mol^{-1}

32. The time for half life period of a certain reaction A → products is 1 hour. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

1) 4 h 2) 0.5 h 3) 0.25 h 4) 1 h

33. $NaNO_3$ on being heated to 500°C gives

1) $NaNO_2$ only 2) $Na_2O + N_2$ 3) $NaNO_2 + O_2$ 4) $NaNO_2 + O_2 + N_2$

34. The structure of diborane (B_2H_6) contains

1) Four 2c - 2e bonds and two 3c - 2e bonds 2) Two 2c - 2e bonds and four 3c - 2e bonds

3) Two 2c - 2e bonds and two 3c - 3e bonds 4) Four 2c - 2e bonds and four 3c - 2e bonds

35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask to 300 K, how many moles of water are in the vapour phase when equilibrium is established?

(Given : Vapour pressure of H_2O at 300 K is 3170 Pa ; R = 8.314 J K^{-1} mol^{-1})

1) 5.56×10^{-3} mol 2) 1.53×10^{-2} mol 3) 4.46×10^{-2} mol 4) 1.27×10^{-3} mol

36. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

1) propene 2) 1-butene 3) 2-butene 4) ethene

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})

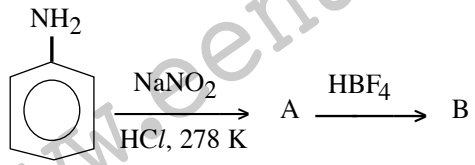
1) 0.0372 K 2) 0.0558 K 3) 0.0744 K 4) 0.0186 K

38. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is

1) 2 - Butanol 2) 2 - Methylpropan - 2 - ol

3) 2 - Methylpropanol 4) 1 - Butanol

39. In the chemical reactions,



the compounds, 'A' and 'B' respectively are

1) nitrobenzene and fluorobenzene

2) phenol and benzene

3) benzene diazonium chloride and fluorobenzene

4) nitrobenzene and chlorobenzene

40. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

1) 59.0 2) 47.4 3) 23.7 4) 29.5

41. The energy required to break one mole of Cl − Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl − Cl bond is

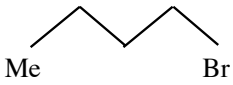
(C = 3×10^8 ms^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})

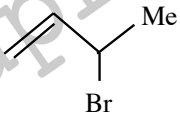
1) 594 nm 2) 640 nm 3) 700 nm 4) 494 nm

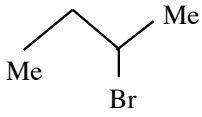
42. The reaction $2\ ZnS + 3O_2 \rightarrow 2\ ZnO + 2\ SO_2$ in the metallurgical process of zinc is called

1) Calcination 2) Cupellation 3) Smelting 4) Roasting

43. Consider the following bromides:

(A) 

(B) 

(C) 

The correct order of SN^1 reactivity is

1) B > C > A 2) B > A > C 3) C > B > A 4) A > B > C

44. Which one of the following has an optical isomer?

1) $[Zn(en)(NH_3)_2]^{2+}$ 2) $[Co(en)_3]^{3+}$

3) $[Co(H_2O)_4(en)]^{3+}$ 4) $[Zn(en)_2]^{2+}$

[en = ethylenediamine)

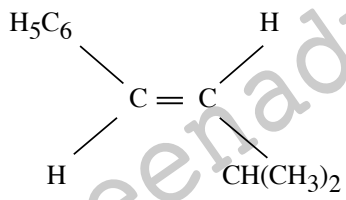
45. Which of the following decarboxylates most easily

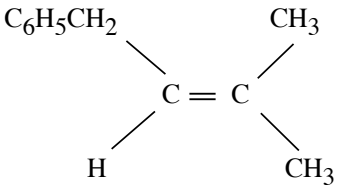
1) Ph – CH_2 – COOH 2) Ph – CO – COOH

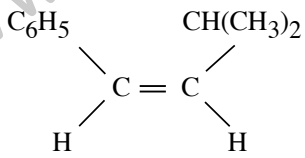
3) Ph – CO – CH_2 – COOH 4) Ph – COOH

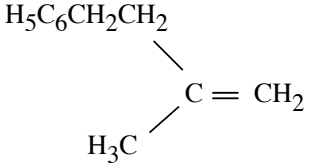
46. The main product of the following reaction is

$C_6H_5\ CH_2\ CH(OH)\ CH(CH_3)_2 \xrightarrow{\text{Conc. } H_2SO_4} ?$

1) 

2) 

3) 

4) 

47. Chlorobenzene on treatment with sodium in dry ether gives diphenyl. The name of the reaction is

1) Fittig reaction 2) Wurtz - Fittig reaction

3) Sandmeyer reaction 4) Gatterman reaction

48. Movement of colloidal particles under the influence of electrostatic field is

1) Electrophoresis 2) Electrolysis 3) Dialysis 4) Ionisation

49. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

1) 288 pm 2) 398 pm 3) 618 pm 4) 144 pm

50. The correct order of increasing basicity of the given conjugate bases (R = CH_3) is

1) $RCO\bar{O} < HC = \bar{C} < \bar{R} < \bar{N}H_2$

2) $\bar{R} < HC \equiv \bar{C} < RCO\bar{O} < \bar{N}H_2$

3) $RCO\bar{O} < \bar{N}H_2 < HC \equiv \bar{C} < \bar{R}$

4) $RCO\bar{O} < HC \equiv \bar{C} < \bar{N}H_2 < \bar{R}$

51. The correct sequence which shows decreasing order of the ionic radii of the elements is

1) $Al^{3+} > Mg^{2+} > Na^{+} > F^{-} > O^{2-}$ 2) $Na^{+} > Mg^{2+} > Al^{3+} > O^{2-} > F^{-}$

3) $Na^{+} > F^{-} > Mg^{2+} > O^{2-} > Al^{3+}$ 4) $O^{2-} > F^{-} > Na^{+} > Mg^{2+} > Al^{3+}$

52. Planar structure is shown by

1) CO_3^{2-} 2) BCl_3 3) $N(SiH_3)_3$ 4) All of these

53. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows:

$$\frac{2}{3}\ Al_2O_3 \longrightarrow \frac{4}{3}\ Al + O_2,\ \Delta_r G = +966\ \text{kJ mol}^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

1) 4.5 V 2) 3.0 V 3) 2.5 V 4) 5.0 V

54. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

1) 9 2) 10 3) 11 4) 8

55. In the balanced chemical reaction,

$IO_3^{-} + aI^{-} + bH^{+} \rightarrow cH_2O + dI_2$, a, b, c and d respectively correspond to

1) 5, 6, 3, 3 2) 5, 3, 6, 3 3) 3, 5, 3, 6 4) 5, 6, 5, 5

56. Out of the following, the alkene that exhibits optical isomerism is

1) 3 – methyl – 2 – pentene 2) 4 – methyl – 1 – pentene

3) 3 – methyl – 1 – pentene 4) 2 – methyl – 2 – pentene

57. Biuret test is not given by

1) carbohydrates 2) polypeptides 3) urea 4) proteins

58. The correct order of $E^{\circ}_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

1) Mn > Cr > Fe > Co 2) Cr > Fe > Mn > Co

3) Fe > Mn > Cr > Co 4) Cr > Mn > Fe > Co

59. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

1) teflon 2) nylon 6, 6

3) polystyrene 4) natural rubber

60. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If Te is the temperature at equilibrium, the reaction would be spontaneous when

1) Te > T 2) T > Te

3) Te is 5 times T 4) T = Te

MATHEMATICS

61. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$ where $0 \le \alpha, \beta \le \frac{\pi}{4}$ then $\tan 2\alpha =$

1) $\frac{56}{33}$ 2) $\frac{19}{12}$ 3) $\frac{20}{7}$ 4) $\frac{25}{16}$

62. Let S be a non-empty subset of R (R is the set of Real Numbers). Consider the following statement:

P: There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P?

1) There is no rational number $x \in S$ such that $x \le 0$

2) Every rational number $x \in S$ satisfies $x \le 0$

3) $x \in S$ and $x \le 0 \Rightarrow$ x is not rational

4) There is a rational number $x \in S$ such that $x \le 0$

63. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ then vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = 0$ and $\vec{a} \cdot \vec{b} = 3$ is

1) $2\hat{i} - \hat{j} + 2\hat{k}$ 2) $\hat{i} - \hat{j} - 2\hat{k}$

3) $\hat{i} + \hat{j} - 2\hat{k}$ 4) $-\hat{i} + \hat{j} - 2\hat{k}$

64. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the X - axis, is

1) y = 1 2) y = 2 3) y = 3 4) y = 0

65. Solution of the differential equation $\cos x\, dy = y(\sin x - y)\, dx$, $0 < x < \frac{\pi}{2}$ is

1) $y \sec x = \tan x + c$ 2) $y \tan x = \sec x + c$

3) $\tan x = (\sec x + c)y$ 4) $\sec x = (\tan x + c)y$

66. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

1) $4\sqrt{2} + 2$ 2) $4\sqrt{2} - 1$ 3) $4\sqrt{2} + 1$ 4) $4\sqrt{2} - 2$

67. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

1) $2x + 1 = 0$ 2) $x = -1$ 3) $2x - 1 = 0$ 4) $x = 1$

68. For all $x \in R$, $x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is

1) $a < -5$ 2) $-5 < a < 2$

3) $a > 5$ 4) $2 < a < 5$

69. Consider the following relations:

R = {(x, y) | x, y are real numbers and x = wy for some rational number w};

S = $\left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| \text{ m, n, p and q are integers such that n, q} \neq 0 \text{ and qm = pn}\right\}$. Then

1) Neither R nor S is an equivalence relation.

2) S is an equivalence relation but R is not an equivalence relation.

3) R and S both are equivalence relations.

4) R is an equivalence relation but S is not an equivalence relation.

70. Let f : R $\rightarrow$ R be defined by

f (x) $= \begin{cases} k - 2x, & \text{if } x \leq -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$. If 'f' has a local minimum at x = –1, then a possible value of k is

1) 0 2) $-\frac{1}{2}$ 3) –1 4) 1

71. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$.

Then the length of the semi - major axis is

1) $\frac{5}{3}$ 2) $\frac{8}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$

Directions: Questions Number 72 to 75 are Assertion - Reason type questions. Each of these questions contains two statements. Statement – 1 **(Assertion)** and Statement – 2 **(Reason).**

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

72. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement - 1: The number of different ways the child can buy the six ice-creams is $^{10}C_5$.

Statement - 2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4B's in a row.

1) Statement – 1 is true, Statement – 2 is false.

2) Statement – 1 is false, Statement – 2 is true.

3) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.

4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement –1.

73. **Statement - 1:** The point A (3, 1, 6) is the mirror image of the point B (1, 3, 4) in the plane x – y + z = 5.

Statement - 2: The plane x – y + z = 5 bisects the line segment joining A (3, 1, 6) and B (1, 3, 4).

1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not the correct explanation for Statement - 1.

2) Statement - 1 is true, Statement - 2 is false

3) Statement - 1 is false. Statement - 2 is true

4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation for Statement - 1.

74. Let $S_1 = \sum_{j=1}^{10} j\,(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$, $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$

Statement - 1: $S_3 = 55 \times 2^9$

Statement - 2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not the correct explanation for Statement - 1.

2) Statement - 1 is true, Statement - 2 is false

3) Statement - 1 is false. Statement - 2 is true

4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation for Statement - 1.

75. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define Tr(1) = sum of diagonal elements of A and |A| = determinant of matrix A.

Statement - 1: Tr(1) = 0

Statement - 2: |A| = 1

1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not the correct explanation for Statement - 1.

2) Statement - 1 is true, Statement - 2 is false

3) Statement - 1 is false. Statement - 2 is true

4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation for Statement - 1.

76. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x-\alpha)^2}$ is equal to

1) 0 2) $\frac{a^2}{2}(\alpha - \beta)^2$ 3) $\frac{1}{2}(\alpha - \beta)^2$ 4) $\frac{-a^2}{2}(\alpha - \beta)^2$

77. For a regular polygon, let 'r' and 'R' be the radii of the inscribed and the circumscribed circles. A false statement among the following is

1) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

2) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

3) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

4) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

78. If $f : R \to R$ is a function defined by $f(x) = [x] \cos\left[\frac{2x-1}{2}\right]\pi$, where [x] denotes the greatest integer function, then f is

1) discontinuous only at non-zero integral values of x

2) continuous only at x = 0

3) continuous for every real x

4) discontinuous only at x = 0

79. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals ..... (where $i = \sqrt{-1}$)

1) 1 2) 2 3) ∞ 4) 0

80. A line AB in three-dimensional space makes angles 45° and 120° with the positive X – axis and the positive Y – axis respectively. If AB makes an acute angle θ with the positive Z – axis, then θ equals

1) 45° 2) 60° 3) 75° 4) 30°

81. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

1) $\sqrt{17}$ 2) $\frac{17}{\sqrt{15}}$ 3) $\frac{23}{\sqrt{17}}$ 4) $\frac{23}{\sqrt{15}}$

82. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in A.P. with common difference –2, then the time taken by him to count all notes is

1) 34 minutes 2) 125 minutes

3) 135 minutes 4) 24 minutes

83. Let f: R $\to$ R be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

1) $\frac{2}{3}$ 2) $\frac{3}{2}$ 3) 3 4) 4

84. Let p(x) be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

1) 21 2) 41 3) 42 4) $\sqrt{41}$

85. Let f: (–1, 1) $\to$ R be a differentiable function with $f(0) = -1$ and $f'(0) = 1$.

Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

1) –4 2) 0 3) –2 4) 4

86. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

1) $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

2) $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

3) $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

4) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

87. Consider the system of linear equations:

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

1) exactly 3 solutions
2) a unique solution
3) no solution
4) infinite number of solutions

88. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colour is

1) $\frac{2}{7}$ 2) $\frac{1}{21}$ 3) $\frac{2}{23}$ 4) $\frac{1}{3}$

89. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

1) $\frac{11}{2}$ 2) 6 3) $\frac{13}{2}$ 4) $\frac{5}{2}$

90. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

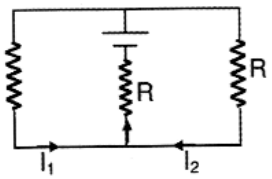
1) $-35 < m < 15$ 2) $15 < m < 65$ 3) $35 < m < 85$ 4) $-85 < m < -35$

KEY

1-2; 2-3; 3-2; 4-3; 5-2; 6-4; 7-1; 8-2; 9-2; 10-3; 11-3; 12-2; 13-3; 14-1; 15-2; 16-2; 17-1; 18-4; 19-3; 20-3; 21-3; 22-3; 23-4; 24-4; 25-3; 26-2; 27-1; 28-1; 29-4; 30-4; 31-2; 32-3; 33-3; 34-1; 35-4; 36-3; 37-2; 38-2; 39-3; 40-3; 41-4; 42-4; 43-1; 44-2; 45-3; 46-1; 47-1; 48-1; 49-4; 50-4; 51-4; 52-4; 53-3; 54-2; 55-1; 56-3; 57-1; 58-1; 59-2; 60-2; 61-1; 62-2; 63-4; 64-3; 65-4; 66-4; 67-2; 68-2; 69-2; 70-3; 71-2; 72-3; 73-3; 74-2; 75-2; 76-2; 77-2; 78-3; 79-1; 80-2; 81-3; 82-1; 83-4; 84-1; 85-1; 86-4; 87-3; 88-1; 89-1; 90-1.

EXPLANATIONS

1. A moving conductor is equivalent to a battery of emf = vBl (motion emf)



Equivalent circuit

$I = I_1 + I_2$

Applying Kirchoff's law

$I_1R + IR - vBl = 0$ .....(1)

$I_2R + IR - vBl = 0$ .....(2)

adding (1) and (2)

$2\,IR + IR = 2\,vBl$

$I = \frac{2vBl}{3R}$

$I_1 = I_2 = \frac{vBl}{3R}$

2. $U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}(q_0 e^{-t/T})^2 = \frac{q_0^2}{2C} e^{-2t/T}$

(where $\tau = CR$)

$U = U_i e^{-2t/\tau}$

$\frac{1}{2} U_i = U_i e^{-2t_1/\tau}$

$\frac{1}{2} = e^{-2t_1/\tau} \qquad \Rightarrow t_i = \frac{T}{2} \ln 2$

Now $q = q_0 e^{-t/T}$

$\frac{1}{4} q_0 = q_0 e^{-t/2T}$

$t_2 = T \ln 4 = 2 T \ln 2$

$\therefore \frac{t_1}{t_2} = \frac{1}{4}$

5. $\rho_{oil} < \rho < \rho_{water}$

Oil is the least dense of them so it should settle at the top with water at the base. Now the ball is denser than oil but less denser than water. So, it will sink through oil but will not sink in water. So it will stay at the oil-water interface.

6. $v = Ky\hat{i} + Kx\hat{j}$

$\frac{dy}{dt} = Ky \quad \frac{dy}{dt} = Kx$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{Kx}{Ky}$

$ydy = xdx$

$y^2 = x^2 + c$

7. The magnetic field in between because of each will be in opposite direction

$B_{\text{in between}} = \frac{\mu_0 i}{2\pi x}\hat{j} - \frac{\mu_0 i}{2\pi (2d - x)}(-\hat{j})$

$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{2d - x}\right](\hat{j})$

at $x = d$, $B_{\text{in Between}} = 0$

for $x < d$, $B_{\text{in Between}} = (\hat{j})$

for $x > d$, $B_{\text{in Between}} = (-\hat{j})$

Towards x net magnetic field will add up and direction will be $(-\hat{j})$

Towards x' net magnetic field will add up and direction will be $(\hat{j})$

8. At t = 0, inductor behaves like an infinite resistance

So at $t = 0$, $i = \frac{v}{R_2}$ and at $t = \infty$, inductor behaves like a conducting wire $i = \frac{V}{R_{eq}} = \frac{V(R_1 + R_2)}{R_1 R_2}$

9. From the graph, it is a straight line so, uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity $= \frac{2}{2} = 1$ m/s

Final velocity $= -\frac{2}{2} = -1$ m/s

$\vec{P_i} = 0.4$ N – s

$\vec{Pj_i} = -0.4$ N – s

$\vec{J} = \vec{P_f} - \vec{P_1} = -0.4 - 0.4$

$= -0.8$ N – s ($\vec{J}$ = impulse)

$|\vec{J}| = 0.8$ N – s

10. After decay, the daughter nuclei will be more stable hence binding energy per nucleon will be more than that of their parent nucleus.

12. In positive beta decay a proton is transformed into a neutron and a positron is emitted.

$p^+ \longrightarrow n0 + e^+$

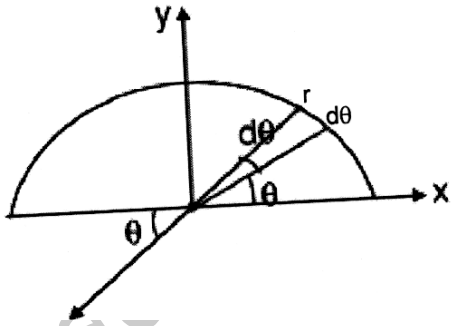
no. of neutrons initially was A – Z

no. of neutrons after decay (A – Z) – 3 × 2 (due to alpha particles) + 2 × 1 (due to positive beta decay)

The no. of proton will reduce by 8. [as 3 × 2 (due to alpha particles) + 2 (due to positive beta decay)]

Hence atomic number reduces by 8.

13. Linear charge density $\lambda = \left(\frac{q}{\pi r}\right)$



$E = \int dE \sin\theta (-\hat{j}) = \int \frac{K.dq}{r^2} \sin\theta (-\hat{j})$

$E = \frac{K}{r^2} \int \frac{qr}{\pi r} d\theta \sin\theta (-\hat{j})$

$= \frac{K}{r^2} \frac{q}{\pi} \int_0^{\pi} \sin d\theta (-\hat{j})$

$= \frac{q}{2\pi^2 \varepsilon 0 r^2} (-\hat{j})$

14. Truth table for given combination is

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

This comes out to be truth table of OR gate.

15. The efficiency of cycle is $\eta = 1 - \frac{T_2}{T_1}$

for adiabatic process $TV^{\gamma - 1}$ = constant

For diatomic gas $\gamma = \frac{7}{5}$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma - 1}$

$\frac{7}{5}-1$

$T_1 = T_2\,(32)$

$= T_2\,(2^5)^{2/5} = T_2 \times 4$

$T_1 = 4T_2$

$\eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$

16. $4 \times 10^3 = 1020 \times hf$

$f = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$

$f = 6.03 \times 10^{16}$ Hz

The obtained frequency lies in the band of X - rays.

18. The given circuit is under resonance as $X_L = X_C$

Hence power dissipated in the circuit is $P = \frac{V^2}{R} = 242$ W

20. $U(x) = \frac{a}{X^{12}} - \frac{b}{X^6}$

$U(x = \infty) = 0$

as, $F = \frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$

at equilibrium, $F = 0$

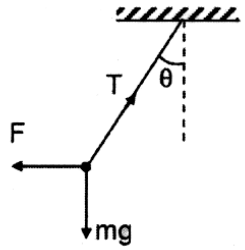
$\therefore x^6 = \frac{2a}{b}$

$\therefore$ Uat equilibrium $= \dfrac{a}{\left(\dfrac{2a}{b}\right)^2} - \dfrac{b}{\left(\dfrac{2a}{b}\right)} = \dfrac{-b^2}{4a}$

$\therefore$ D = [U (x = ∞) – Uat equilibrium] $= \dfrac{b^2}{4a}$

21. From F.B.D. of sphere, using Lami's theorem

$\dfrac{F}{mg} = \tan\theta$ .....(i)



when suspended in liquid, as θ remains same

$\therefore \dfrac{F'}{mg\left(1 - \dfrac{\rho}{d}\right)} = \tan\theta$ .....(ii)

using (i) and (ii)

$\dfrac{F}{mg} = \dfrac{F'}{mg\left(1 - \dfrac{\rho}{d}\right)}$ where, $F' = \dfrac{F}{K}$

$\therefore \dfrac{F}{mg} = \dfrac{F}{mg\,K\left(1 - \dfrac{\rho}{d}\right)}$

Or $K = \dfrac{1}{1 - \dfrac{\rho}{d}} = 2$

23. $S = t^3 + 5$

$\therefore$ speed, $v = \dfrac{ds}{dt} = 3t^2$

and rate of change of speed = $\dfrac{dv}{dt} = 6t$

$\therefore$ tangential acceleration at t = 2s, $at = 6 \times 2 = 12\ m/s^2$

at t = 2s, v = 3(2)2 = 12 m / s

$\therefore$ centripetal acceleration, $ac = \dfrac{v^2}{R} = \dfrac{144}{20}\ m/s^2$

$\therefore$ net acceleration $\sqrt{a_t^2 + a_i^2}$

$\approx 14\ m / s^2$

24. $mg \sin\theta = ma$

$\therefore\ a = g \sin\theta$

where a is along the inclined plane

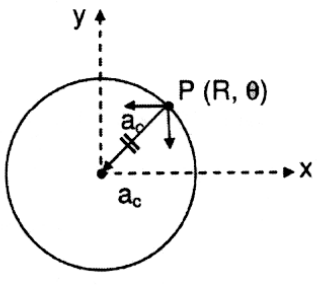
$\therefore$ vertical component of acceleration is $g \sin^2\theta$

$\therefore$ relative vertical acceleration of A with respect to B is

$g[\sin^2 60 - \sin^2 30] = \frac{g}{2} = 4.9\ m/s^2$ in vertical direction.

25. For a particle in uniform circular motion,

$\vec{a} = \frac{v^2}{R}$ towards centre of circle



$\therefore \vec{a} = \frac{v^2}{R}(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$

Or

$\vec{a} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

30. $T = \mu v^2 = \mu\ \frac{\omega^2}{k^2} = 0.04\ \frac{(2\pi / 0.004)^2}{(2\pi / 0.50)^2}$

$= 6.25$ N

31. Enthalpy of formation of NH_3

$= -46$ kJ / mole

$\therefore\ N_2 + 3H_2 \longrightarrow 2NH_3$

$\Delta H_f = -2 \times 46$ kJ mol

Bond breaking is endothermic and Bond formation is exothermic

Assuming 'x' is the bond energy of N – H bond (kJ mol^{-1})

$\therefore\ 712 + (3 \times 436) - 6x = -46 \times 2$

$\therefore\ x = 352$ kJ / mol

32. For a zero order reaction $k = \frac{x}{t}$ ..... (1)

Where x = amount decomposed

k = zero order rate constant

for a zero order reaction

$$k = \frac{[A]_0}{2t_{\frac{1}{2}}} \quad \text{..... (2)}$$

Since $[A_0] = 2M$, $t_{1/2} = 1$ hr; $k = 1$

$\therefore$ from equation (1)

$$t = \frac{0.25}{1} = 0.25 \text{ hr}$$

35. $n = \frac{PV}{RT} = 128 \times 10^{-5}$ moles

$$\frac{3170 \times 10^{-5} \text{ atm} \times 1 \text{ L}}{0.0821 \text{ L atm k}^{-1} \text{ mol}^{-1} \times 300\text{K}}$$

$\approx 1.27 \times 10^{-3}$ mol

36. 2 – butene is symmetrical alkene

$$CH_3 — CH = CH — CH_3 \xrightarrow[Zn / H_2O]{O_3} 2.CH_3CHO$$

Molar mass of CH_3CHO is 44 u

37. Vant Hoff's factor (i) for $Na_2SO_4 = 3$

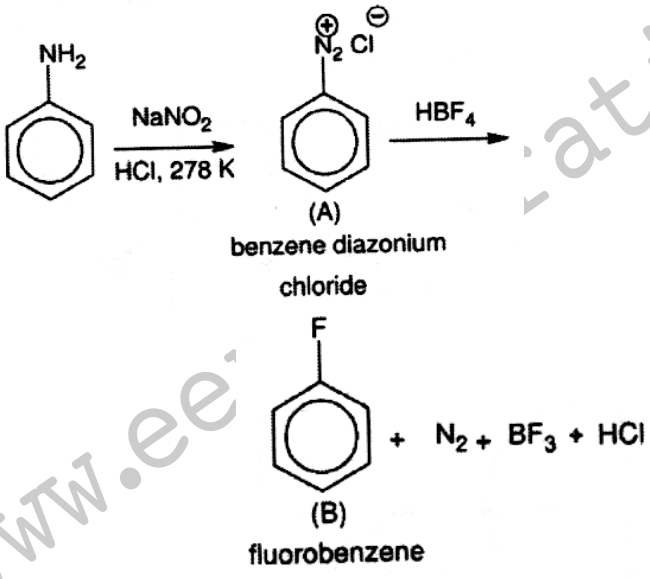
$\therefore \Delta T_f = (i) k_f m$

$= 3 \times 1.80 \times \frac{0.01}{1} = 0.0558$ K

38. 3° alcohols react fastest with $ZnCl_2$ / Conc. HCl due to formation of 3° carbocation and

$\therefore$ 2 – methyl propan – 2 – ol is the only 3° alcohol.

39.



40. Moles of HCl reading with ammonia

= (moles of HCl absorbed) – (moles of NaOH solution required)

$= (20 \times 0.1 \times 10^{-3}) - (15 \times 0.1 \times 10^{-3})$

= moles of NH_3 evolved.

= moles of nitrogen in organic compound

$\therefore$ wt. of nitrogen in org. comp

$= 0.5 \times 10^{-3} \times 14$

$= 7 \times 10^{-3}$ g

$$\% \text{ wt} = \frac{7 \times 10^{-3}}{29.5 \times 10^{-3}} = 23.7\%$$

41. Energy required for 1 Cl_2 molecule $= \frac{242 \times 10^3}{NA}$ Joules.

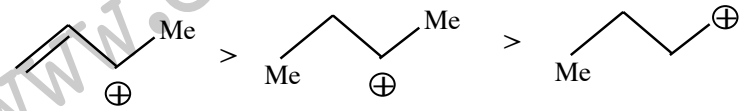
This energy is contained in photon of wavelength 'λ'

$$\frac{hc}{\lambda} = E \Rightarrow \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

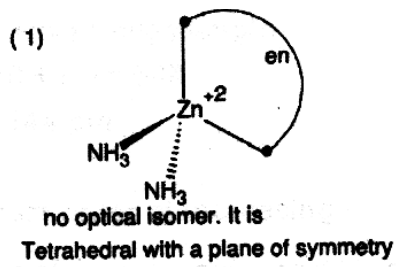
$$= \frac{242 \times 10^3}{6.022 \times 1023}$$

$\lambda = 4947\ \mathring{A} \approx 494$ nm

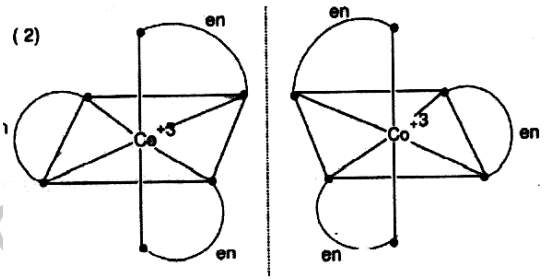
43. SN^1 proceeds via carbocation intermediate, the most stable one forming the product faster. Hence reactivity order for A, B, C depends on stability of carbocation created.



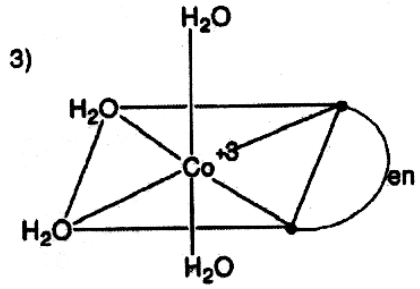
44. Only option (2) is having non-super imposable mirror image and hence one optical isomer



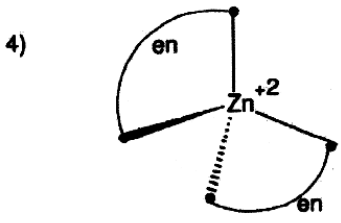
no optical isomer. It is
Tetrahedral with a plane of symmetry



optical isomer



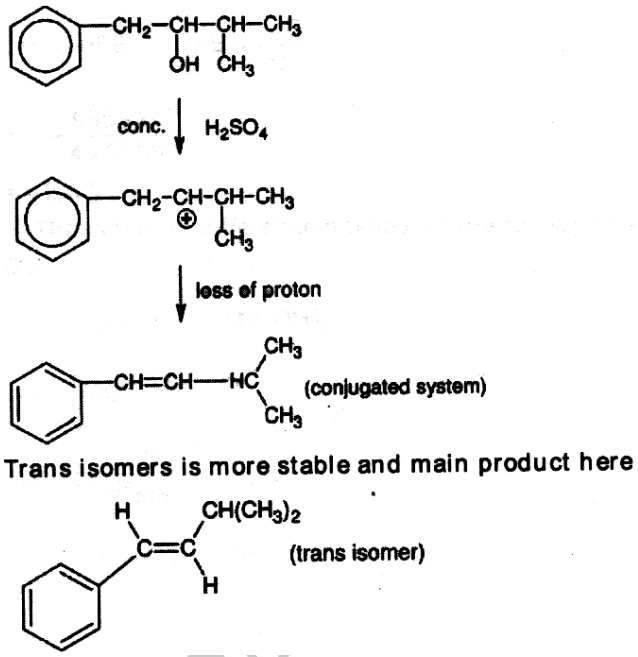
Horizontal plane is plane of symmtry



no optical isomer, it is
tetrahedral with a plane of symmetry

45. $X_{Heptane} = 0.45$.

46.



Trans isomers is more stable and main product here

49. For an ionic substance in FCC arrangement,

$2 (r^+ + r^-) =$ edge length

$2(110 + r^-) = 508$

$r^- = 144$ pm

50. Correct order of increasing basic strength is

$R - COO^{(-)} < CH \equiv C^{(-)} < NH_2^{(-)} < R^{(-)}$

51. For isoelectronic species higher the $\frac{Z}{e}$ ratio, smaller the ionic radius

$\frac{Z}{e}$ for $O^2 = \frac{8}{19} = 0.8$

$F^- = \frac{9}{10} = 0.9$

$Na^+ = \frac{11}{10} = 1.1$

$Mg^{2+} = \frac{12}{10} = 1.2$

$Al^{3+} = \frac{13}{10} = 1.3$

53. $\Delta G = -nFE \Rightarrow E = \frac{-\Delta G}{nF}$

$E = -\frac{966 \times 10^3}{4 \times 96500} = -2.5$ V

$\therefore$ The potential difference needed for the reduction = 2.5 V

54. $Mg^{2+} + 2OH^- \rightleftharpoons Mg(OH)_2$

$K_{sp} = [Mg^{2+}][OH^-]^2$

$[OH^-] = \sqrt{\frac{K_{sp}}{[Mg^{2+}]}} = 10^{-4}$

$\therefore$ pOH = 4 and pH = 10

56. $H_2C = HC - \underset{CH_3}{\overset{H}{|}} - C_2H_5$

only 3-methyl-1-pentene has a chiral carbon.

57. It is a test characteristic of amide linkage. Urea also has amide linkage like proteins.

59. nylon 6,6 is a polymer of adipic acid and hexamethylene diamine

$$-\left(-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-NH-(CH_2)_6-NH-\right)_n$$

60. $\Delta G = \Delta H - T\Delta S$

at equilibrium, $\Delta G = 0$

for a reaction to be spontaneous ΔG should be negative

$\therefore T > T_e$

61. $\cos(\alpha+\beta) = \frac{4}{5} \Rightarrow \tan(\alpha+\beta) = \frac{3}{4}$

$\sin(\alpha-\beta) = \frac{5}{13} \Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$

$$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta) = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\frac{5}{12}} = \frac{56}{33}$$

62. P: there is a rational number

$x \in S$ such that $x > 0$

$\sim P$: Every rational number $x \in S$ satisfies $x \le 0$

63. $\vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow \vec{b} \cdot \vec{c} = 0$

$\Rightarrow (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}).(\hat{i} - \hat{j} - \hat{k}) = 0$

$b_1 - b_2 - b_3 = 0$

and $\vec{a} \cdot \vec{b} = 3$

$\Rightarrow b_2 - b_3 = 3$

$b_1 = b_2 + b_3 = 3 + 2b_3$,

$\vec{b} = (3 + 2b_3)\hat{i} + (3 + b_3)\hat{j} + b_3\hat{k}$.

64. Parallel to X - axis

$\Rightarrow \frac{dy}{dx} = 0 \qquad \Rightarrow 1 - \frac{8}{x^3} = 0$

$\Rightarrow x = 2 \quad \Rightarrow y = 3$

Equation of tangent is

$y - 3 = 0\ (x - 2) \quad \Rightarrow y - 3 = 0$

65. $\cos x\, dy = y(\sin x - y)\, dx$

$\frac{dy}{dx} = y \tan x - y^2 \sec x$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y} \tan x = \sec x$

Let $\frac{1}{y} = t$

$-\frac{1}{y2}\frac{dy}{dx} = \frac{dt}{dx}$

$-\frac{dy}{dx} - t \tan x = -\sec x$

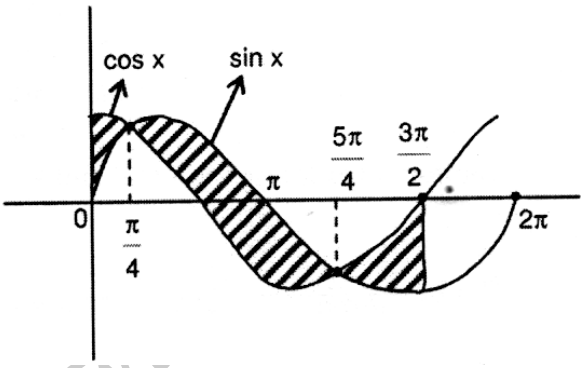
$\Rightarrow \frac{dt}{dx} + (\tan x)\, t = \sec x$

I.F. $= e^{\int \tan x\, dx} = \sec x$

Solution is t (I.F.) $= \int$ (I.F.) sec x dx

$\frac{1}{y} \sec x = \tan x + c$

66. $\int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\cos x - \sin x)dx + \int_{5\pi/4}^{3\pi/2} (\sin x - \cos x) = 4\sqrt{2} - 2$



67. The locus of perpendicular tangents is directrix

i.e., $x = -a$; $x = -1$

69. xRy need not implies y Rx

S: $\frac{m}{n}$ s $\frac{p}{q} \Leftrightarrow qm = pn$

$\frac{m}{n}$ s $\frac{m}{n}$ reflexive

$\frac{m}{n}$ s $\frac{p}{q}$ $\qquad$ $\frac{p}{q}$ s $\frac{m}{n}$ symmetric

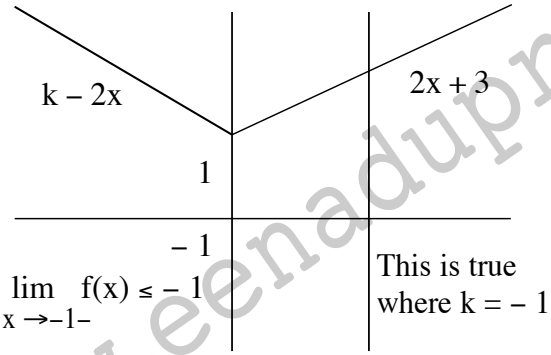
$\frac{m}{n}$ s $\frac{p}{q}$, $\frac{p}{q}$ s $\frac{r}{s}$

$\Rightarrow$ qm = pn, ps = rq

$\Rightarrow$ ms = rn transitive

70. f(x) = k – 2x $\qquad$ if $x \le 1$

= 2x + 3 $\qquad$ if $x > -1$



$\lim_{x \to -1-} f(x) \le -1$

This is true where k = – 1

73. A = (3, 1, 6); B = (1, 3, 4)

Mid-point of AB = (2, 2, 5) lies on the plane

and d.r's of AB = (2, –2, 2)

d.r's Of normal to plane = (1, –1, 1).

AB is not perpendicular bisector

$\therefore$ A is not image of B.

Statement-2 is correct but it is not correct explanation.

74. $S_1 = \sum_{j=1}^{10} j\ (j-1)\ \frac{10!}{j(j-1)(j-2)!(10-j)!}$

$= 90 \sum_{j=2}^{10} j\ \frac{8!}{(j-2)!(8-(j-2))!} \rightarrow 90.2^8$

$S_2 = \sum_{j=1}^{10} j\ \frac{10!}{j(j-1)(9-(j-1))!}$

$= 10 \sum_{j=1}^{10} \frac{9!}{(j-1)!(9-(j-1))!} \rightarrow 10.2^9$

$S_3 = \sum_{j=1}^{10} \left[j\,(j-1) + j \right] \frac{10!}{j!(10-j)!}$

$= \sum_{j=1}^{10} j\,(j-1)\ {}^{10}C_j = \sum_{j=1}^{10} j\ {}^{10}C_j$

$= 90.2^8 + 10.2^9$

$= 90.2^8 + 20.2^8$

$= 110.2^8 = 55.2^9$

75. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $abcd \neq 0$

$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} . \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\Rightarrow A^2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix}$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$

$ab + bd = ac + cd = 0$

$c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0$

$\text{Trace}A = a + d = 0$

$|A| = ad - bc = -a^2 - bc = -1.$

77. $r = \frac{a}{2} \cot \frac{\pi}{n}$

'a' is side of polygon.

$R = \frac{a}{2} \text{cosec} \frac{\pi}{n}$

$\frac{r}{R} = \frac{\cot \frac{\pi}{n}}{\text{cosec} \frac{\pi}{n}} = \cos \frac{\pi}{n}$

$\cos \frac{\pi}{n} \neq \frac{2}{3}$ for any $n \in N$.

79. Let $z = x + iy$

$|z - 1| = |z + 1| \Rightarrow \text{Re } z = 0 \Rightarrow x = 0$

$|z - 1| = |z - i| \Rightarrow x = y$

$|z + 1| = |z - i| \Rightarrow y = -x$

Only (0, 0) will satisfy all conditions.

$\Rightarrow$ Number of complex number z = 1

80. $l = \cos 45° = \frac{1}{\sqrt{2}}$

$m = \cos 120° = -\frac{1}{2}$

$n = \cos \theta$

where θ is the angle which line makes with positive Z - axis.

Now $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2 \theta = 1$

$\cos\theta = \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{1}{2}$ (θ Being acute)

$\Rightarrow \theta = \frac{\pi}{3}$

81. Slope of line L $= -\frac{b}{5}$

Slope of line K $= -\frac{3}{c}$

Line L is parallel to line k.

$\Rightarrow \frac{b}{5} = \frac{3}{c} \quad \Rightarrow bc = 15$

(13, 32) is a point on L.

$\Rightarrow \frac{13}{5} + \frac{32}{b} = 1 \qquad \Rightarrow \frac{32}{b}$

$\Rightarrow b = -20$

$\Rightarrow c = -\frac{3}{4}$

Equation of K ; $y - 4x = 3$

Distance between L and K

$= \frac{|52 - 32 + 3|}{\sqrt{17}} = \frac{23}{\sqrt{17}}$

82. Till 10^{th} minute number of counted notes $= 1500$

$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)]$

$= n[148 - n + 1]$

$n^2 - 149n + 3000 = 0$

$n = 125$ is not possible.

Total time $= 24 + 10$

$= 34$ minutes.

83. $f(x)$ is a positive increasing function

$\Rightarrow 0 < f(x) < f(2x) < f(3x)$

$\Rightarrow 0 < 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

$\Rightarrow \lim_{x\to\infty} 1 \le \lim_{x\to\infty} \frac{f(2x)}{f(x)} \le \lim_{x\to\infty} \frac{f(3x)}{f(x)}$

By sandwich theorem

$\Rightarrow \lim\limits_{x\to\infty} \dfrac{f(2x)}{f(x)} = 1$

84. $p'(x) = p'(1 - x)$

$\Rightarrow p(x) = -p(1 - x) + c$

at $x = 0$

$p(0) = -p(1) + c \quad \Rightarrow 42 = c$

now $p(x) = -p(1 - x) + 42$

$\Rightarrow p(x) + p(1 - x) = 42.$

$$I = \int_0^1 p(x)\,dx = \int_0^1 p(1 - x)\,dx$$

$$2I = \int_0^1 (42)\,dx \quad \Rightarrow I = 21.$$

85. $g'(x) = 2(f(2f(x) + 2)) \left(\dfrac{d}{dx}(f(2f(x) + 2))\right)$

$= 2f(2f(x) + 2)\, f'(2f(x) + 2) \,.\, (2f'(x))$

$\Rightarrow g'(0) = 2f(2f(0) + 2) \,.\, f'(2f(0) + 2) \,.\, 2(f'(0))$

$= 4f(0)\, f'(0)$

$= 4\,(-1)\,(1) = -4.$

87. $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0$

$D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution.

88. $n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

Probability $= \dfrac{3 \times 4 \times 2}{{}^9C_3} = \dfrac{24 \times 3!}{9!} \times 6!$

$= \dfrac{24 \times 6}{9 \times 8 \times 7} = \dfrac{2}{7}$

89. $\sigma_x^2 = 4$

$\sigma_y^2 = 5$

$\overline{x} = 2$

$\overline{y} = 4$

$\frac{\Sigma x_i}{5} = 2 \qquad \Sigma x_i = 10; \quad \Sigma y_i = 20;$

$\sigma_x^2 = \left(\frac{1}{5}\Sigma x_i^2\right) - (\bar{x})^2$

$= \sigma_x^2 = \frac{1}{5}\Sigma y_i^2 - (\bar{y})^2$

$\Sigma x_i^2 = 40$

$\Sigma y_i^2 = 105$

$\sigma_z^2 = \frac{1}{10}\left(\Sigma x_i^2 + \Sigma y_i^2\right) - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$

$= \frac{1}{10}(40 + 105) - 9 = \frac{145 - 90}{10} = \frac{55}{10} = \frac{11}{2}$

90. Circle $x^2 + y^2 - 4x - 8y - 5 = 0$

Centre = (2, 4).

Radius = $\sqrt{4 + 16 + 5} = 5$

If circle is intersecting line $3x - 4y = m$

at two distinct points.

$\Rightarrow$ length of perpendicular from centre < radius

$\Rightarrow \frac{|6 - 16 - m|}{5} < 5$

$\Rightarrow |10 + m| < 25$

$\Rightarrow -25 < m + 10 < 25$

$\Rightarrow -35 < m < 15$

(ఈ నమూనా ప్రశ్నపత్రాన్ని శ్రీగాయత్రి విద్యాసంస్థలకు చెందిన నిపుణులు రూపొందించారు)