JEE

MATHEMATICS

1. If $\bar{a} = \frac{1}{\sqrt{10}}(3i + k)$ and $\bar{b} = \frac{1}{7}(2i + 3j - 6k)$ then the value of $(2\bar{a} - \bar{b}).[(\bar{a}\ , \bar{b})\ ,(\bar{a} + 2\bar{b})]$

1) 5 2) 3 3) −5 4) −3

2. If the coefficients of x^{-2} and x^{-4} is the expansion of $\left(x^{\frac{1}{3}} + \frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, x > 0 are m and n respectively then $\frac{m}{n}$ is equal to

1) 27 2) 182 3) $\frac{5}{4}$ 4) $\frac{4}{5}$

3. If z and w are two non zero complex numbers such that $|zw| = 1$ and $\text{Arg}(z) - \text{Arg}(w) = \frac{\Pi}{2}$, then $\bar{z}w$ is equal to

1) −1 2) 1 3) −i 4) i

4. Negation of "Paris is in France and Landon is in England" is

1) Paris in England and Landon is in France

2) Paris is not in France or Landon is not in England

3) Paris is in England or Landon is not in France

4) Paris is not in England and Landon is not in France

5. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $\left(P^2 + Q^2\right)$ is equal to

1) −2 2) 1 3) 0 4) −1

6. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,\text{adj}A = AA^T$ then 5a + b is equal to

1) 13 2) −1 3) 5 4) 4

7. The set S = {1, 2, 3.....12} is to partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \varnothing$ the number of ways to partition S is

1) $\frac{12!}{3!\,(4!)^3}$ 2) $\frac{12!}{3!\,(3!)^4}$ 3) $\frac{12!}{(4!)^3}$ 4) $\frac{12!}{(3!)^4}$

8. A husband and wife appear in an interview for two vacancies in the same post. The probability of husbands selection is $\frac{1}{7}$ and that of wife is $\frac{1}{5}$. The probability that only one of them will be selected is

1) $\frac{1}{35}$ 2) $\frac{24}{35}$ 3) $\frac{2}{7}$ 4) $\frac{2}{35}$

9. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle and a point with in a circle is choosen at random then the probability that this point lies outside of the ellipse is

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) $\frac{8}{9}$ 4) $\frac{2}{5}$

10. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha_n - \beta_n$, $n \geq 1$ then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is equal to

1) 6 2) −6 3) 3 4) −3

11. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777..... is

1) $\frac{7}{81}\left(179 - 10^{-20}\right)$ 2) $\frac{7}{9}\left(99 - 10^{-20}\right)$

3) $\frac{7}{81}\left(179 + 10^{-20}\right)$ 4) $\frac{7}{9}\left(99 + 10^{-20}\right)$

12. If the standard deviation of the numbers 2, 3, a and 11 is 3.5 then which of the following is true?

1) $3a^2 - 23a + 44 = 0$ 2) $3a^2 - 26a + 55 = 0$

3) $3a^2 - 32a + 84 = 0$ 4) $3a^2 - 34a + 91 = 0$

13. $\cos(\theta - \alpha) = a$ and $\cos(\theta - \beta) = b$ then $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta) =$

1) $a^2 + b^2$ 2) $a^2 - b^2$ 3) $b^2 - a^2$ 4) $-a^2 - b^2$

14. The number of points of intersection of the two curves $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is

1) 0 2) 1 3) 2 4) ∞

15. The period of $f(x) = [x] + [2x] + [3x] + ... + [nx] - \dfrac{n(n+1)}{2}x$, where $n \in N$ is

1) 1 2) 2 3) 4 4) 6

16. $\int e^{\sin x}(x\cos x - \tan x \sec x)dx$

1) $e^{\sin x}(x - \sec x) + c$ 2) $e^{\sin x}(x + \sec x) + c$

3) $e^{\sin x}(x + \cos x) + c$ 4) $e^{\sin x}(x - \cos x) + c$

17. $\displaystyle\int_0^{\pi/4} \frac{x^2dx}{(x\sin x + \cos x)^2}$

1) $\dfrac{4+\pi}{4-\pi}$ 2) $\dfrac{4+\pi}{2(4-\pi)}$ 3) $\dfrac{4-\pi}{4+\pi}$ 4) $2\left[\dfrac{4-\pi}{4+\pi}\right]$

18. The area (in sq.units) of the region described by $\{(x, y) : y^2 \leq 2x, \text{ and } y \geq 4x - 1\}$ is

1) $\frac{7}{32}$ 2) $\frac{5}{64}$ 3) $\frac{15}{64}$ 4) $\frac{9}{32}$

19. The locus of the feet of perpendiculars drawn from the point (a, 0) on tangents to the circle $x^2 + y^2 = a^2$ is

1) $a^2\left(x^2 + y^2 + ax\right)^2 = a^2\left(y^2 + (x + a)^2\right)$

2) $a^2\left(x^2 + y^2 - ax\right)^2 = y^2 + (x - a)^2$

3) $\left(x^2 + y^2 - ax\right)^2 = a^2\left[y^2 + (x - a)^2\right]$

4) $a^2\left[(x^2 + y^2) - a^2x^2\right] = \left[y^2 + (x - a)^2\right]$

KEY

1-3; 2-2; 3-3; 4-2; 5-3; 6-3; 7-3; 8-3; 9-1; 10-3; 11-3; 12-3; 13-1; 14-1; 15-1; 16-1; 17-3; 18-4; 19-3.

HINTS & SOLUTIONS

1. $\bar{a} = \frac{1}{\sqrt{10}}(3\bar{i} + \bar{k})$ $\bar{b} = \frac{1}{7}\left(2\bar{i} + 3\bar{j} - 6\bar{k}\right)$

$|\bar{a}| = |\bar{b}| = 1, \bar{a} \cdot \bar{b} = 0$

$\Rightarrow |\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin 90° = 1$

$\left[2\bar{a} - \bar{b} \quad \bar{a} \times \bar{b} \quad \bar{a} + 2\bar{b}\right]$

$= \left(\bar{a} \times \bar{b}\right) . \left[\left(\bar{a} + 2\bar{b}\right) \times \left(2\bar{a} - \bar{b}\right)\right]$

$= \left(\bar{a} \times \bar{b}\right) . 5 \left(\bar{b} \times \bar{a}\right)$

$= -5 (a \times b)^2 = -5 (1) = -5$

2. $T_{r+1} = {}^{18}C_r \left(x^{1/3}\right)^{18-r} \left(\frac{1}{2x^{1/3}}\right)^r$

$= {}^{18}C_r \left(\frac{1}{2}\right)^r x^{\frac{18-2r}{3}}$

For coefficient of x^{-2}, $\frac{18-2r}{3} = -2 \Rightarrow r = 12$

For coefficient of x^{-4}, $\frac{18-2r}{3} = -4 \Rightarrow r = 15$

$$\frac{m}{n} = \frac{{}^{18}C_{12}\left(\frac{1}{2}\right)^{12}}{{}^{18}C_{15}\left(\frac{1}{2}\right)^{15}} = \frac{{}^{18}C_6 (2)^3}{{}^{18}C_3} = 182$$

3. $|zw| = 1 \Rightarrow |z| = \frac{1}{|w|}$

$\arg z - \arg w = \frac{\Pi}{2} \Rightarrow \arg\left(\frac{z}{w}\right) = \frac{\Pi}{2}$

$\frac{z}{w} = \left|\frac{z}{w}\right|\left(\cos\frac{\Pi}{2} + i \sin\frac{\Pi}{2}\right)$

$\frac{z}{w} = |z|^2 i \Rightarrow \bar{z} w = -i$

4. Let p : Paris in France

q : Landon is in England

We have $p \wedge q$

Its negation $\sim (p \wedge q) = \sim p \vee \sim q$

i.e Paris is not in France or Landon is not in England.

5. $(P^2 + Q^2)(P - Q) = P^3 - P^2Q + PQ^2 - Q^3 = 0$

If $\det(P^2 + Q^2) \neq 0 \Rightarrow P^2 + Q^2$ is invertible

And we get $P - Q = 0 \Rightarrow P = Q$

Which is a contradiction $\Rightarrow \det(P^2 + Q^2) = 0$

6. $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

$= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 3b + 10a \end{bmatrix}$

$15a = 2b \Rightarrow b = \frac{15a}{2}$, $10a + 3b = 13$,

$10a + \frac{45a}{2} = 13$

$65a = 26$, $a = \frac{2}{5}$, $b = \frac{15}{2} \times \frac{2}{5} = 3$

$5a + b = 2 + 3 = 5$

7. No. of ways is $^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 = \frac{12!}{(4!)^3}$

8. $P(A) = \frac{1}{7}$; $P(B) = \frac{1}{5}$

Required probability

$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= P(A).P(\overline{B}) + P(\overline{A}).P(B)$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{2}{7}$

9. Probability $= 1 - \frac{\pi ab}{\pi a^2} = 1 - \frac{\pi a^2 \sqrt{1 - e^2}}{\pi a^2}$

$= 1 - \frac{1}{3} = \frac{2}{3}$

10. Given $x^2 - 6x - 2 = 0$

$\Rightarrow x^2 = 6x + 2$

$\Rightarrow \alpha^2 = 6\alpha + 2$

$\Rightarrow \alpha^{10} = 6\alpha^9 + 2\alpha^8$ .... (1)

And $\beta^{10} = 6\beta^9 + 2\beta^8$ .... (2)

(2) − (1) $\Rightarrow$ $a^{10} = 6a_9 + 2a_8$

$\frac{a_{10} - 2a_8}{2a_9} = 3$

11. $\frac{7}{9}\left[0.9 + 0.99 + 0.999 + ... + 20 \text{ terms}\right]$

$\frac{7}{9}\left[(1 - 0.1) + (1 - 0.01) + (1 - 0.0001) + ... + 20 \text{ terms}\right]$

$\frac{7}{9}\left[20 - \left\{\frac{1}{10} + \left(\frac{1}{10}\right)^2 + ... + 20 \text{ terms}\right\}\right]$

$= \frac{7}{9}\left[20 - \frac{1}{0} \;\frac{\left\{1 - \left(\frac{1}{10}\right)^{20}\right\}}{1 - \frac{1}{10}}\right]$

$= \frac{7}{9}\left[\frac{179 + 10^{-20}}{9}\right] = \frac{7}{81}\;[179 + 10^{-20}]$

12. 2, 3, a, 11 $\rightarrow$ S.D = 3.5

$(3.5)^2 = -\left(\frac{16 + a}{4}\right)^2 + \frac{134 + a^2}{4}$

$\Rightarrow \frac{49}{4} = -\frac{(16 + a)^2}{16} + \frac{134 + a^2}{4}$

$\Rightarrow 3a^2 - 32a + 84 = 0$

13. $\cos(\alpha - \beta) = \cos[(\theta - \beta) - (\theta - \alpha)]$

$= ba + \sqrt{1 - a^2}\;\sqrt{1 - b^2}$

$[\cos(\alpha - \beta) - ab]^2 = (1 - a^2)(1 - b^2)$

$a^2 + b^2 = \sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$

14. $y = 5x^2 + 2x + 3 = 5\left[x^2 + \frac{2}{5}x + \frac{3}{5}\right]$

$= 5\left[\left(x + \frac{1}{5}\right)^2 + \frac{3}{5} - \frac{1}{25}\right]$

$= 5\left(x + \frac{1}{5}\right)^2 + \frac{14}{5} > 2$

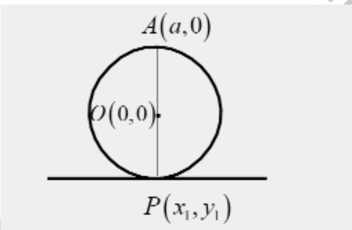
Since $y = 2\sin x \le 2$, so there cannot be any point of intersection.

15. $[x] + [2x] + ... + [nx] - (x + 2x + ... + nx)$

$= -(\{x\} + \{2x\} + ... + \{nx\})$

Period of f(x) = Lcm of

$\left(1, \frac{1}{2}, \frac{1}{3} ...... + \frac{1}{n}\right) = 1$



Eq of the tangent is $y = mx + a\sqrt{1 + m^2}$

$\Rightarrow (y - 0) = -\frac{1}{m}(x - a) \Rightarrow m = \frac{a - x}{y}$

Substitute in tangent equation.

16. $\int x\, e^{\sin x} \cos x\, dx - \int e^{\sin x} \sec x \tan x\, dx$

$= x\, e^{\sin x} - \int e^{\sin x}\, dx - \int e^{\sin x} \sec x - \int e^{\sin x} \cos x \sec x\, dx]$

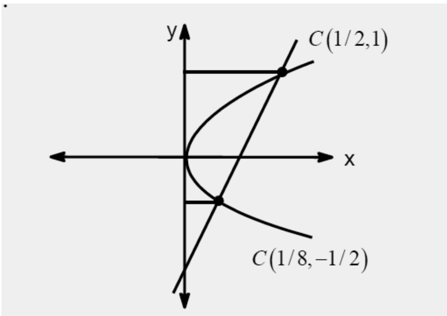
$= x\, e^{\sin x} - \int e^{\sin x}\, dx - e^{\sin x} \sec x + \int e^{\sin x}\, dx$

$= e^{\sin x} (x - \sec x) + c$

17. $\int_0^{\pi/4} \frac{x \cos x}{(x \sin x + \cos x)^2} \cdot \frac{x}{\cos x}\, dx$

$f(x) = \frac{x}{\cos x}$, $g(x) = \frac{x \cos x}{(x \sin x + \cos x)^2}$

18.



Given curves are $y^2 = 2x$ & $y = 4x - 1$

Point of intersections is $\left(\frac{1}{2}, 1\right); \left(\frac{1}{8}, -\frac{1}{2}\right)$

$\int_{-1/2}^{1} \left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy = \frac{9}{32}$