# JEE-ADVANCED

## MODEL GRAND TEST PAPER - I

Time : 3 hrs ] [Number of questions : 60

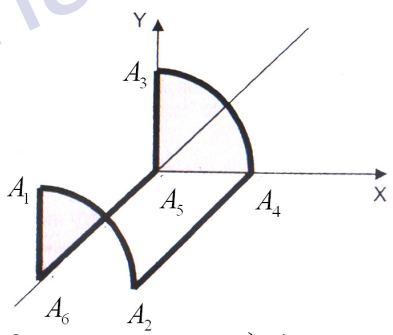
## PHYSICS

**Section-I**

**Single Correct Answer Type**

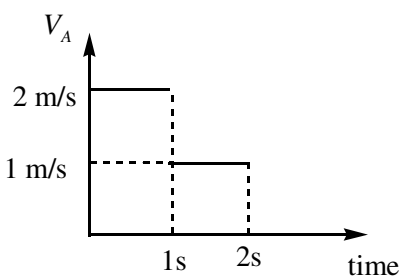
This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

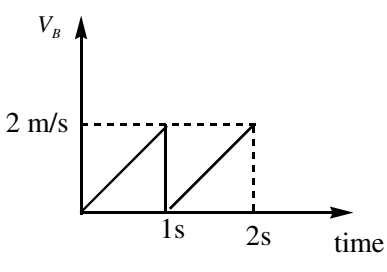
1. A time varying magnetic field $B = B_0 t\hat{k}$ is confined in a cylindrical region. cutting the XY plane on a circle of radius $x^2 + y^2 = 4$. We have placed a wire frame as shown. Segment $A_1A_2$ and $A_3A_4$ are identical quarter circles. The net emf induced in the wire frame is equal to




a) zero b) $2B_0$ c) $4B_0$ d) $B_0$

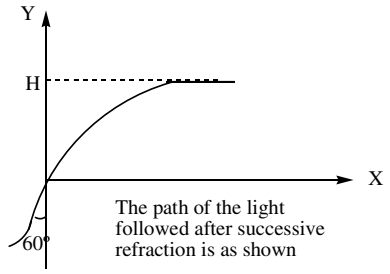
2. Two particles A and B start moving from the same point on the X-axis. The velocity versus time graph for the particle is as shown in figure. The maximum relative separation between the two particles will be equal to




a) $\frac{1}{2}m$ b) $\frac{3}{4}m$ c) $\frac{5}{4}m$ d) None

3. A ray of light is incident at the origin at an angle 60° with Y-axis as shown in figure. The refractive index is a function of y according to the relation $\mu = \frac{2}{1+y^2}$. What is the value of H, shown in figure?




a) $\sqrt{\frac{2}{\sqrt{3}}-1}$ b) $\sqrt{\frac{2}{\sqrt{3}}}$ c) $\sqrt{\left(\sqrt{3}-1\right)}$ d) None

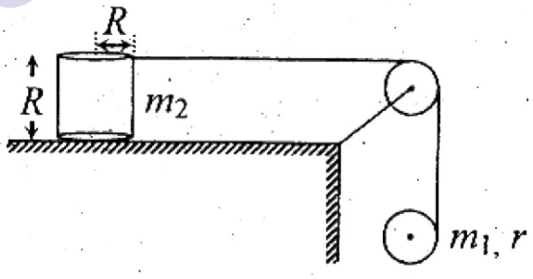
4. Dimensionally power of lens is equivalent to :

a) $\frac{E\sqrt{LC}}{B}$ b) $\frac{E}{B\sqrt{LC}}$ c) $\frac{B\sqrt{LC}}{E}$ d) $\frac{B}{E\sqrt{LC}}$

5. The potential in certain region is given as $V = 2x^2$, then the charge density of that region is

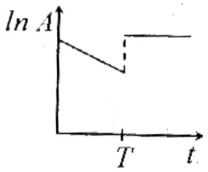
a) $-\frac{4x}{\varepsilon_0}$ b) $-\frac{4}{\varepsilon_0}$ c) $-4\varepsilon_0$ d) $-2\varepsilon_0$

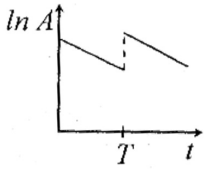
6. A disc of mass $m_1$ radius r is released from rest in the fig shown. If the cylinder is on the verge of slipping as well as toppling then coefficient of friction between cylinder and table surface is :

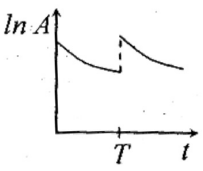


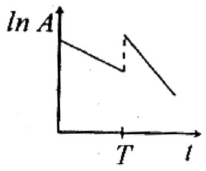

a) 1 b) $\frac{1}{2}$ c) 3 d) $\frac{1}{3}$

7. At time t = 0, some radioactive gas is injected into a sealed vessel. At time T, some more of the same gas is injected into the same vessel. The graph representing the variation of the logarithm of the activity A of the gas with time t is

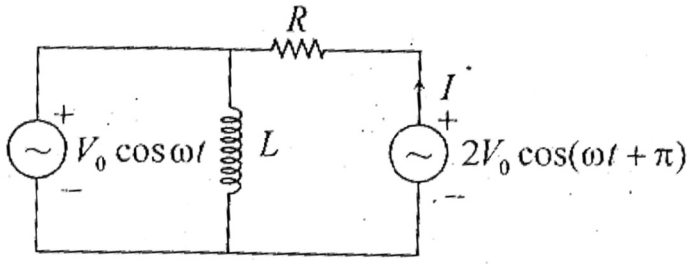
a) 


b) 


c) 


d) 

8. The diagram shows an AC circuit with two voltage sources of same frequency. Find out the value of current I shown in the fig.



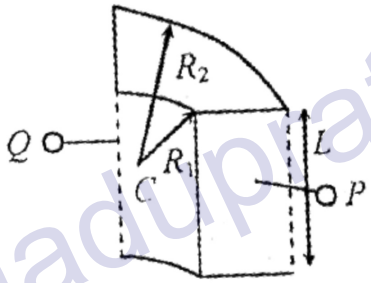

a) $I = \dfrac{V_0 \cos(\omega t)}{R}$

b) $I = \dfrac{V_0 \cos(\omega t + \pi/2)}{R}$

c) $I = \dfrac{V_0 \cos \omega t}{R}$

d) $I = \dfrac{-3V_0 \cos \omega t}{R}$

9. A resistor is formed in the shape of a hollow, quarter cylinder from a material of resistivity $\rho$. The length of cylinder is L, inner and outer radii are $R_1$ and $R_2$ respectively. The resistance of this resistor between the shown terminals P and Q is




a) $\dfrac{2\pi\rho L}{(R_2^2 - R_1^2)}$

b) $\dfrac{2\rho\pi}{L\, ln\left(\dfrac{R_2}{R_1}\right)}$

c) $\dfrac{\pi\rho}{2L\, ln\left(\dfrac{R_2}{R_1}\right)}$

d) $\dfrac{\rho\pi R_1}{L(R_2 - R_1)}$

10. Standing waves are set up in a string of length 240 cm clamped horizontally at both ends. The separation between any two consecutive points where displacement amplitude is $3\sqrt{2}$ cm is 20 cm. The standing waves were set by two traveling waves of equal amplitude of 3 cm. The overtone in which the string is vibrating will be

a) $2^{nd}$

b) $3^{rd}$

c) $4^{th}$

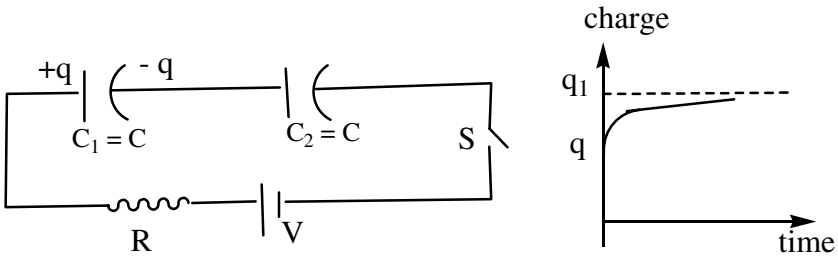
d) $5^{th}$

## Section-II

## (One or More options correct Type)

This section contains 5 multiple choice question. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

11. In the diagram shown, both the capacitors have the same capacitance C. One capacitor has charge +q and other is uncharged. The switch is closed at time t =0. The graph shows the variation of the charge on $C_1$ as a function of time. Then



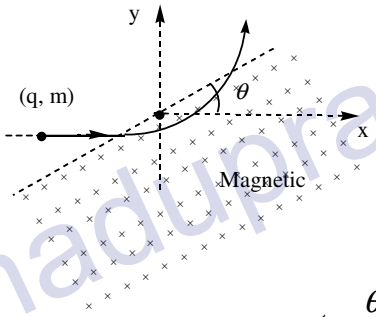

a) Initial value of current in the circuit is $\frac{\left|V-\frac{q}{C}\right|}{R}$

b) Charge present in second capacitor in steady state is $\frac{CV-q}{2}$

c) Value of $q_1$ in steady state is $\frac{CV+q}{2}$

d) Heat energy will be generated in the circuit on closing the switch

12. A uniform magnetic field $-B_0\hat{k}$ exists to the right of the plane $y = x\tan\theta$ as shown. At t = 0 a particle of mass m and positive charge q with velocity $v_0\hat{i}$ enters in magnetic field at origin. Then



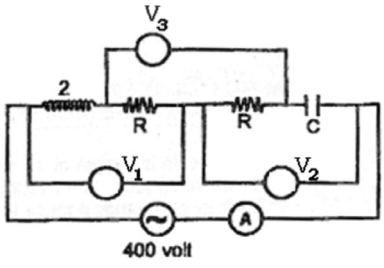

a) particle will come out from magnetic field after $t=\frac{\theta m}{qB_0}$

b) particle will come out from magnetic field after time $t=\frac{2\theta m}{qB_0}$

c) Co-ordinate of point from which particle will come out is $\left[\frac{mv_0}{qB_0}\sin 2\theta, \frac{mv_0}{qB_0}(1-\cos 2\theta), 0\right]$

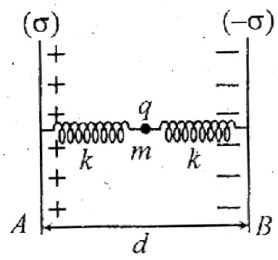
d) Co-ordinate of point from which particle will come is $\left[\frac{mv_0}{qB_0}\sin\theta, \frac{mv_0}{qB_0}(1-\cos\theta), 0\right]$

13. In the shown circuit reading of voltmeter $V_1$ and $V_3$ are 300 volt each then choose correct option/options if reading of ammeter is 10A :




a) $V_2 = 300$ V  b) $V_2 = 400$ V  c) $R = 10\Omega$  d) $R = 20\Omega$

14. Two large on conducting plates having surface charge densities $+\sigma$ and $-\sigma$, respectively, are fixed 'd' distance apart. A small test charge q of mass m is attached to two non – conducting identical springs of spring constant k as shown in the adjacent fig. The charge q is now released from rest with springs in natural length. Then q will [neglect gravity]




a) perform SHM with angular frequency $\sqrt{\frac{2k}{m}}$

b) perform SHM with amplitude $\frac{\sigma q}{2k\in_0}$

c) not perform SHM, but will have a periodic motion if charges are removed on plates as well as on m

d) remain stationary

15. A satellite close to the earth is in orbit above the equator with a period of rotation 1.5hours. If it is above a point P on the equator at some time, it will be above P again after time

a) 1.5 hours b) 1.6 hours if it is rotating from west to east

c) 24/17 hours if it is rotating from east to west

d) 24/17 hours if it is rotating from west to east

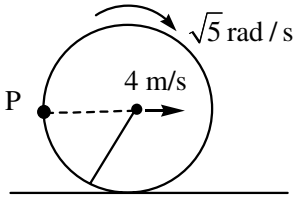
## Section-III

## (Integer value correct Type)

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive)

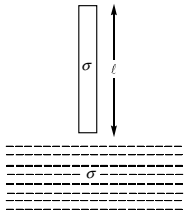
16. A metal rod AB of length L rotates with a constant angular velocity $\omega$ about an axis passing through O and normal to its length. The magnitude of emf induced between ends A and B in the absence of external magnetic field is $\frac{mL^2\omega^2}{ke}$. Hence m is mass of electron and e is charge on electron. Find the value of k.

17. The centre of mass of a disc of radius $\frac{8}{\sqrt{5}}m$ is moving with a velocity of 4 m/s on a horizontal plane. The angular velocity of the disc about it's centre is $\sqrt{5}\,\text{rad/s}$. Find the radius of curvature of the point 'P shown in the figure in meter?



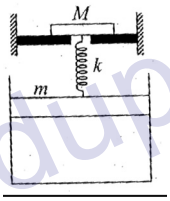
18. A uniform vertical cylinder is released from rest when its lower end just touches the liquid surface of a deep lake. Calculate maximum displacement of cylinder (in meter).

Take $\ell = 4m$ *and* $\frac{\sigma}{\rho} = \frac{1}{2}$




19. A sample of radioactive nuclide $A^{150}$ is having half life 2 hours and it produce $B^{146}$ after emitting $\alpha$ particle. Initially in sample only A was present having mass 50 gm. After four hours difference in mass of sample (A + B) is x gm then value of x is.

20. 0.01 moles of an ideal diatomic gas is enclosed in an adiabatic cylinder of cross–sectional area $A = 10^{-4} m^2$. In the arrangement shown, a block of mass M = 0.8 kg is placed on a horizontal support, and another block of mass m = 1kg is suspended from a spring of stiffness constant k = 16 N/m. Initially, the spring is relaxed and the volume of the gas is $V = 1.4\ 10^{-4}\ m^3$. What is the angular frequency (in $rad.s^{-1}$) of the suspended system ? ($P_0 = 10^5$ N/$m^2$, g = 10 m/$s^2$)






# CHEMISTRY

## Section-I

### Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

21. Standard enthalpy and standard entropy of vaporization of water are + 40 kJ $mol^{-1}$ and + 120 J $mol^{-1}$ $K^{-1}$ respectively. Vapour pressure of water at 27°C expressed as $ln\, P_{H_2O}$ will be (consider Standard temp. to be 300K)

A) – 1.6 B)1.6 C) – 6.1 D) – 3.2

22. Copper I chloride (Cu$C\ell$) used in fireworks emits blue light of wave length 450 nm when heated to high temperature. What is the increment of energy that is emitted at this wave length?

A) $44\times10^{-23} J$ B) $44\times10^{-23} kJ$ C) $4.4\times10^{-23} J$ D) $4.4\times10^{-23} kJ$

23. In an alloy of Mn – Si, atoms of Mn are at 50% of corners and that of Si are at remaining 50% corners of a primitive cubic crystal lattice. If Mn also occupies cubical void, atomic % of Mn in the alloy will be

A) 25% B) 50% C) 75% D) 33.33%

24. For the cell representation $Ag_{(s)} \mid AgBr_{(s)} \mid KBr_{(aq)C_1} \parallel KCl_{(aq)C_2} \mid AgCl_{(s)} \mid Ag_{(s)}$ ; which of the following facts are TRUE?

P: It is an example of concentration cell, if the concentration of metal ion used in both the half cells is identical

Q: This cell can also be represented as $Ag \mid Ag^+\left(\frac{(K_{sp})_{AgBr}}{Br^-}\right) \parallel Ag^+\left(\frac{(K_{sp})_{AgCl}}{Cl^-}\right) \mid Ag$

R: It is an example of electrolytic concentration cell

S: The cell reaction would be spontaneous if $(K_{sp})_{AgBr} \times [Cl^-] < (K_{sp})_{AgCl} \times [Br^-]$

A) P B) R C) QS D) PQRS

25. Aqueous solution of which of the following halide is oxidising

A) $PCl_3$ B) $NCl_3$ C) $PCl_5$ D) $AsCl_3$

26. Which of the following pair of salts forms precipitate with excess of $NH_4OH$

A) $CuSO_4, CdSO_4$ B) $Cu(NO_3)_2, ZnSO_4$

C) $FeCl_3, AlCl_3$ D) all the above

27. I: $\left[\underline{Co}(H_2O)_6\right]^{+3}$ ; II : $\left[\underline{Cu}(H_2O)_6\right]^{+2}$ ; III : $\left[\underline{PtF_6}\right]^{-2}$ IV : $\left[\underline{Co}(H_2O)_3F_3\right]$ The correct statement about the hybridisation of underlined atom of above complexes :

A) I, II, III, IV $\rightarrow sp^3d^2$ B) I, II, III, IV $\rightarrow d^2sp^3$

C) I & III $\rightarrow d^2sp^3$ D) I & IV

28. Consider the following sequence of reactions. The products (X) and (Y), respectively, are: $C_2H_5Cl \xrightarrow[DMSO,heat]{KCN} X \xrightarrow[prolonged\ heat]{H_3O^+} Y$

A) $C_2H_5CN$ and $C_2H_5CH_2NH_2$ B) $C_2H_5CN$ and $C_2H_5CONH_2$

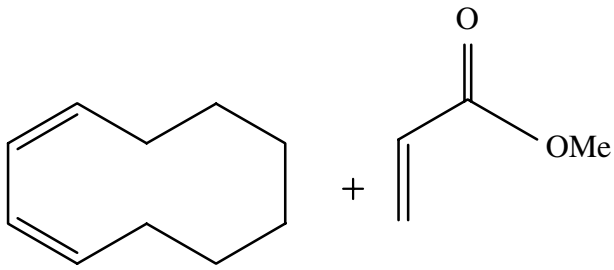
C) $C_2H_5NC$ and $C_2H_5NHCH_3$ D) $C_2H_5CN$ and $C_2H_5COOH$

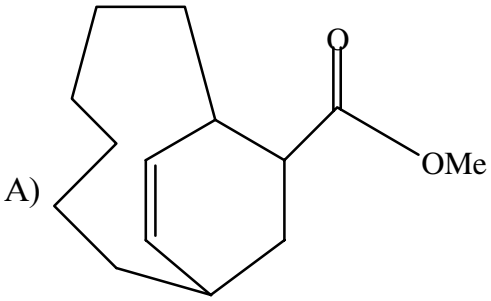
29. Consider the following reactions. Their final product C is:

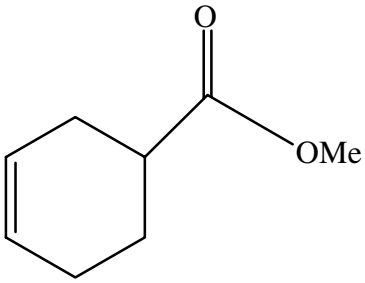
$$Ph_3P + CH_3CH_2Br \longrightarrow A:,\ A + C_2H_5ONa \longrightarrow B:,\ \begin{matrix} CH_3 \\ \diagdown \\ \diagup \\ CH_3 \end{matrix} C=O + B \longrightarrow C$$

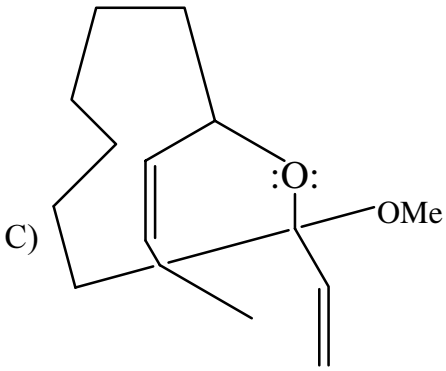
A) $Ph_3\overset{+}{P}CH_2CH_3Br^-$ B) $Ph_3P=CH-CH_3$

C) $\begin{matrix} CH_3 \\ \diagdown \\ \diagup \\ CH_3 \end{matrix} C=CH-CH_3$ D) $\begin{matrix} CH_3 \\ \diagdown \\ \diagup \\ CH_3 \end{matrix} CH-CH=CH_2$

30.  'X'. What should be '$X$'?

A) 

B) 

C) 

D) The reaction will not proceed.



## Section-II

### (One or More options correct Type)

This section contains 5 multiple choice question. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

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31. Which of the following is/are CORRECT?

A) One mole of a liquid mixture containing one mole each of liquid A and liquid B is vaporized. Total vapour pressure is found to be $P_T = \sqrt{P_A^o P_B^o}$

B) A solution of HC$l$ in water forms higher boiling azeotrope which can be distilled with same composition.

C) A 0.1 M solution of $HgCl_2$ in water freezes at – 0.186°C.It concludes that does not ionise in water

D) Ideal solutions can be distilled to separate the pure components.

32. Which of the following is/are TRUE?

A) In acid medium forms a positively charged colloidal sol containing $[SnO_2]Sn^{4+}$

B) In basic medium $SnO_2$ forms a negatively charged colloidal sol containing $[SnO_2]SnO_3^{2-}$

C) is amphoteric in nature

D) neither reacts with acid nor with base

33. Which of the Hydrohalic acid cannot form acidic salt

A) HF B) HCl C) HBr D) HI

34. In which of the following reaction t-butylbenzene is formed?

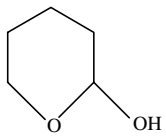
A) Benzene + iso-butyl chloride, $AlCl_3$

B) Benzene + $(CH_3)_2C=CH_2 \xrightarrow{BF_3.HF}$

C) Benzene + t-butyl alcohol $\xrightarrow{H_2SO_4}$

D) Benzene + $(CH_3)_2C=CH_2 \xrightarrow{AlCl_3}$

35. Which of the following compound will give positive Tollens test?

A) $CH_3CHO$ B) 


C) $CH_3CH_2CH_2CH_2\underset{\underset{OH}{|}}{CH}-OCH_3$ D) $CH_3-CH\begin{matrix}\diagup OCH_3\\ \diagdown OCH_3\end{matrix}$

## Section-III

## (Integer value correct Type)

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive)

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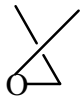
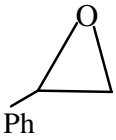
36. For the reaction $2NO+H_2 \rightarrow N_2O+H_2O$, value of $\left(-\frac{dp}{dt}\right)$ changes from 1.5 torr $s^{-1}$ to 0.25 torr $s^{-1}$ when pressure is changed from 360 torr to 150 torr with respect to $NO$ keeping pressure of hydrogen to be constant. If pressure of $NO$ is kept constant, it is observed that value of $\left(-\frac{dp}{dt}\right)$ changes from 1.6 torr $s^{-1}$ to 0.8 torr $s^{-1}$ with respect to pressure of hydrogen from 300 torr to 150 torr. Order of the reaction will be $[\log 2=0.301;\log 3=0.477;\log 5=0.699;\log 7=0.845]$

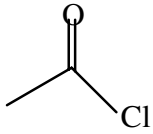
37. The no. of stereo isomers of $\left[Pt(gly)_3\right]^+$ is _____

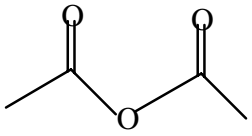
38. Among $NF_3$, $CCl_4, SF_6, PCl_5, SiF_4$, $BCl_3$. The no. of halides which can't under go hydrolysis at ordinary conditions ___

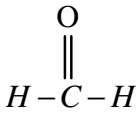
39. How many of the following would produce a tertiary alcohol on treatment with $CH_3CH_2MgBr$?

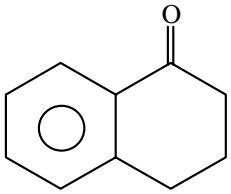
(i) 

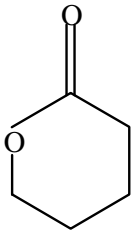
ii) 


iii) 


iv) 


v) 


vi) 


vii) $CH_3 - C \equiv N$ viii) $CO_2$ ix) 


40. How many enentiomeric forms are possible for glucose molecule?

## MATHEMATICS

### Section-I

**Single Correct Answer Type**

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. Range of the function $f(x) = \frac{1}{3^{\{x\}} - 9^{\{x\}} + 1}$ is (where $\{.\}$represents fractional part function)

a) $[1,2]$ b) $[1,2)$ c) $(1,2]$ d) none of these

42. A circle touches two of the smaller sides of $\Delta ABC$ and has its centre on the greatest side where a > b > c. If r is the radius of the incircle and $r_1$ is the radius of the given circle, then

a) $0 < \frac{r_1}{r} < 1$ b) $1 < \frac{r_1}{r} < 2$ c) $2 < \frac{r_1}{r} < 3$ d) $3 < \frac{r_1}{r} < 4$

43. If $f : [0,1] \to (0,\infty)$ then the number of mappings f such that $\int_0^1 f(x)dx = 1$, $\int_0^1 xf(x)dx = \alpha$ and $\int_0^1 x^2 f(x)dx = \alpha^2$ is

a) 2 b) 1 c) 0 d) Infinite

44. Five numbers out of 1, 2, 3,.....,9 are written randomly at four vertices and centroid of a regular tetrahedron. The probability that 6 is written at centroid of tetrahedron is

a) 5/9 b) 4/9 c) 8/9 d) 1/9

45. Let A, B, C, D be distinct points on a circle with centre at O. If there exists non zero real numbers x and y such that $\left|x\overrightarrow{OA}+y\overrightarrow{OB}\right|=\left|x\overrightarrow{OB}+y\overrightarrow{OC}\right|=\left|x\overrightarrow{OC}+y\overrightarrow{OD}\right|=\left|x\overrightarrow{OD}+y\overrightarrow{OA}\right|$ then

a) ABCD is a rectangle
b) ABCD is a square
c) ABCD is a rhombus
d) nothing can be said

46. The number of values of k for which $\left(x^2-(k-2)x-2k\right)\left(x^2+kx+2k-4\right)$ is a perfect square is

a) 1
b) 2
c) 0
d) none of these

47. The values of a and b which satisfy

$\int\frac{2\sin 2x-\cos x}{4-\cos^2 x-4\sin x}dx=a\log\left|\sin x-1\right|+b\log\left|\sin x-3\right|+c$, are respectively

a) $-\frac{3}{2},\frac{11}{2}$
b) $\frac{3}{2},\frac{11}{2}$
c) $\frac{3}{2},-\frac{11}{2}$
d) $-\frac{3}{2},-\frac{11}{2}$

48. The number of natural numbers less than or equal to 2012, which are relatively prime to 2012 is

a) 1004
b) 1006
c) 1005
d) 4

49. Let O be an interior point of $\Delta ABC$ such that $\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}=\vec{0}$. Then the ratio of area of to area of is

a) 2
b) 3/2
c) 3
d) 5/3

50. Let $p(x)=x^5+x^2+1$ has zeroes $x_1,x_2,x_3,x_4,x_5$ and $g(x)=x^2-2$. Then the product $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5)$ is equal to

a) 1/23
b) 23
c) -23
d) none of these

## Section-II

## (One or More options correct Type)

This section contains 5 multiple choice question. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

---

51. Let $f(x)=\left|x+1\right|\left(\left|x\right|+\left|x-1\right|\right)$. Then which of the following is/are true?

a) $f'(-1)=3$
b) f is continuous at $x=-1$
c) f is not differentiable at x = 0
d) $Rf'(1)=5$

52. Let $a_1<a_2<a_3<a_4<a_5<a_6$

$p=a_1+a_2+a_3+.....+a_6$

$q=a_1a_3+a_3a_5+a_5a_1+a_2a_4+a_4a_6+a_6a_2$

$r=a_1a_3a_5+a_2a_4a_6$. Then the equation $2x^3-px^2+qx-r=0$ has

a) one root between $(a_1,a_2)$
b) two roots between $(a_1,a_3)$
c) two roots between $(a_1,a_4)$
d) two roots between $(a_3,a_5)$

53. If $f:N\to\left[-\sqrt{2},\sqrt{2}\right]$ *such that* $f(x)=\sin x+\cos x$, then f(x) is

a) one-one b) onto c) many-one d) into

54. A hyperbola has center C and one focus at P(6,8). If it's two directrices are 3x + 4y + 10 = 0 and 3x+4y-10 = 0, then

a) CP = 10 b) eccentricity= $\sqrt{5}$

c) CP = 8 d) eccentricity = /2

55. If n different objects are distributed at random among n +2 persons so that each person can get any number of things (i.e.,0,1,2,..., n things) then the probability that

a) exactly 2 persons will get none of the objects is $\frac{(n+1)!}{2.(n+2)^{n-1}}$

b) exactly 3 persons will get none of the objects is $\frac{{}^{n+2}C_3\,(n-1)\left({}^{n}C_2\right)(n-2)!}{(n+2)^n}$

c) exactly 3 persons will get none of the objects is $\frac{n.(n-1)^2.(n+1)}{12(n+2)^{n-1}}$

d) exactly 2 persons will get none of the objects is $\frac{n^2.(n-1)^2}{(n+2)^{n-1}}$

## Section-III

## (Integer value correct Type)

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive)

56. The least positive integral value of real $\lambda$ so that the equation $(x-a)(x-c)(x-e)+\lambda(x-b)(x-d)=0,(a>b>c>d>e)$ has distinct real roots is_____

57. Let $z_1,z_2,.....,z_n$ be equi-modular non-zero complex numbers such that $z_1+z_2+.....+z_n=0$.

Then $\mathrm{Re}\left(\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{z_j}{z_k}\right)$ is equal to_____

58. The number of distinct terms in the expansion of $\left(x+y+z+\frac{1}{xy}+\frac{1}{yz}+\frac{1}{xz}\right)^2$ is m and that in the expansion of $\left(x+y+z+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)^2$ is n, then $|m-n|=$_____

59. Let A be a $3\times3$ matrix with real entries. If $AA^T=I$, then the value of $\det(A^2-I_3)=$_____

60. Number of divisors of a natural number n is 105 and n is divisible by exactly 3 distinct prime numbers and is having the least value. The number of divisors of 4k +1 form (where $k\in N$) of the number n is ____



***

# JEE-ADVANCED

## MODEL GRAND TEST PAPER - I

## KEY SHEET

### PHYSICS

01) A 02) C 03) A 04) D 05) C 06) A 07) B 08) D 09) C 10) D

11) ABCD 12) BC 13) BD 14) AB 15) BC 16) 4 17) 5 18) 4

19) 1 20) 6

### CHEMISTRY

21) A 22) B 23) C 24) C 25) B 26) C 27) C 28) D 29) C 30) A

31) ABCD 32) ABC 33) BCD 34) ABC 35) ABC 36) 3 37) 4

38) 3 39) 5 40) 2

### MATHEMATICS

41) D 42) B 43) C 44) D 45) B 46) A 47) A 48) A 49) C 50) C

51) BC or BCD 52) A,C 53) A,D 54) A,B 55) AB 56) 1 57) 0 58) 1

59) 0 60) 8

# JEE-ADVANCED

## MODEL GRAND TEST PAPER - I

### HINTS & SOLUTIONS

### PHYSICS

1. A

Due to time varying magnetic field, emf will induce only in segment $A_1A_2$ and $A_3A_4$ only. But Net emf in the loop will be zero. Hence current zero.

2. C

3. A

Using snell's law between origin (y = 0) y = H, we get,

$$2\times\sin 60^\circ = \frac{2}{1+H^2}.\sin 90^\circ$$

$$\frac{\sqrt{3}}{2} = \frac{1}{1+H^2} \Rightarrow 1+H^2 = \frac{2}{\sqrt{3}}; H = \sqrt{\frac{2}{\sqrt{3}}-1}$$

4. D

The dimension of $\frac{B}{E\sqrt{LC}}$ *is* $\frac{1}{meter}$

$$\left(\frac{B}{E}=\frac{1}{V}\&\frac{1}{\sqrt{LC}}=\omega=\frac{1}{T}\right)$$

5. C

$$E = -\frac{dV}{dx} = -4x$$

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_0} = -4$$

$$\Rightarrow \rho = -4\varepsilon_0$$

6. A

$m_1g - T = m_1a$

$$T_r = m_1\frac{r^2}{2}\frac{a}{r}$$

$$\Rightarrow a = \frac{2g}{3}; T = \frac{m_1g}{3}$$

Also, $T = \mu m_2 g$ , $\Rightarrow \mu = \frac{m_1}{3m_2}$

Balancing torque about A, we get $TR = m_2gR$

$$\frac{m_1g}{3}, R = m_2gR$$

$$\Rightarrow m_1 = 3m_2 \qquad \Rightarrow \mu = \frac{m_1}{3m_2} = 1$$

7. B

$A = \lambda N = \lambda N_0 e^{-\lambda t}$

$\ln A = \ln \lambda N_0 - \lambda t$

8. D

Applying KVL in the outer loop

$V_0 \cos \omega t + IR - 2V_0 \cos(\omega t + \pi) = 0$

$\Rightarrow IR = 2V_0 \cos(\omega t + \pi) - V_0 \cos \omega t$

$\Rightarrow I = \dfrac{-2V_0 \cos \omega t - V_0 \cos \omega t}{R} = \dfrac{-3V_0 \cos \omega t}{R}$

9. A

$(B_1)_x = 0,\ (B_1)_y = \dfrac{\mu_0 I}{4\pi d}(-\hat{j})$

$(B_2)_x = \dfrac{\mu_0 I}{8\pi d}\sin\theta \hat{i},\ (B_2)_y = \dfrac{\mu_0 I}{8\pi d}\cos\theta(-\hat{j})$

$(B_3)_x = \dfrac{\mu_0 I}{8\pi d}\cos\theta(-\hat{i}),\ (B_3)_y = \dfrac{\mu_0 I}{8\pi d}\sin\theta(-\hat{j})$

$B_{net} = \dfrac{\mu_0 I}{8\pi d}[\sin\theta - \cos\theta]\hat{i} - \dfrac{\mu_0 I}{8\pi d}(2 + \cos\theta + \sin\theta)\hat{j}\backslash$

$B_{net} = \dfrac{\mu_0 I}{8\pi d}\sqrt{6 + 4\sin\theta + 4\cos\theta}$

$f(\theta) = 6 + 4\sin\theta + 4\cos\theta$

$f'(\theta) = 4(\cos\theta - \sin\theta) = 0$

$\tan\theta = 1 \ for\ 0^0 < \theta < \dfrac{\pi}{2}$

$\theta = \dfrac{\pi}{4}$

10. A

Given that $P = 2e^{2V}$

$\dfrac{nRT}{V} = 2e^{2v}$

$T = \dfrac{2V}{nR}e^{2V}$

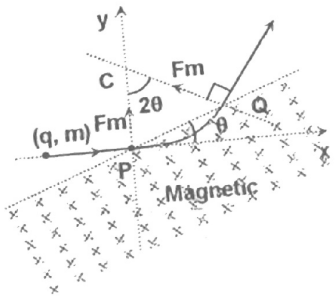
By differentiating $\dfrac{dT}{dV} = \dfrac{2}{nR}\left[2Ve^{2v} + e^{2V}\right]$

$\dfrac{dT}{dV} = \dfrac{2}{nR}\left[2Ve^{2v} + e^{2V}\right]$

11. ABCD

12. BC

First find centre of circular path and than use

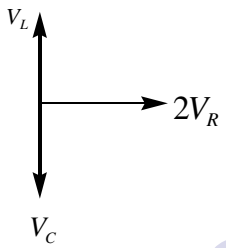



$$PC = PQ = \frac{mv}{qB}$$

$$t = \frac{arc\ length}{v}$$

13. BD

$$V_1 = \sqrt{V_L^2 + V_R^2} = 300$$




$$V_2 = \sqrt{V_C^2 + V_R^2} = 300$$

$$V_L = V_C$$

$$2V_R = 400$$

IR = 200

$$\therefore R = 20\,\Omega$$

14. AB

A = deformation in equilibrium state

$$2kA = \frac{\sigma}{\in_0} q\ ,$$

$$\therefore A = \frac{\sigma q}{2k \in_0}$$

Springs are connected in parallel $k_{eq} = 2k$

Angular frequency $= \sqrt{\frac{2k}{m}}$

15. BC

When rotating west to east

$$\omega_{rel} = \frac{2\pi}{1.5} - \frac{2\pi}{24} = 2\pi\left(\frac{15}{24}\right)$$

$$T' = \frac{2\pi}{1.5} + \frac{24}{15} = 1.6 hrs$$

When rotating east to west

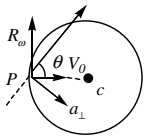
$$\omega_{rel} = \frac{2\pi}{1.5} + \frac{2\pi}{24} = 2\pi\left(\frac{17}{24}\right)$$

$$\Rightarrow T' = \frac{2\pi}{\omega_{rel}} = \frac{24}{17} hrs$$

16. 4

17. 5

velocity of Point 'P $\sqrt{V_0^2 + (R\omega_0)^2}$




$$V_P = 4\sqrt{5}\, m/s$$

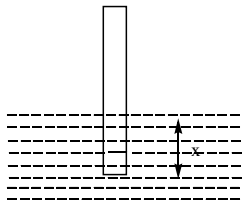
$$a_{centripetal} = \frac{(8)^2}{8} = 8\sqrt{5}\, m/s^2$$

and $a_\perp = a_{cp} \sin\theta = 16\, m/s^2$

$$R = \frac{(V_P)^2}{a_\perp} = 5\, m$$

18. 4

$$Al\sigma g - Ax\rho g = Al\sigma a$$




$$a = b - \frac{\rho g x}{\sigma \ell}$$

$$\int_0^v v.dv = \int_0^x g - \frac{\rho g x}{\sigma \ell} dx \qquad \Rightarrow \frac{v^2}{2} = gx - \frac{\rho g}{\sigma \ell}\frac{x^2}{2}$$

At maximum displacement,

$$\Rightarrow x = \frac{2\sigma\ell}{\rho} = 2 \times \frac{1}{2} \times 4 = 4m$$

19. 1

$A^{150} \rightarrow B^{146} + \alpha \ particle$

at t = 4 hour = $2t_{1/2}$

$m_A = \frac{50}{4} gm \ and \ m_B = \frac{146}{4} gm$

Now difference of mass $(A+B) = 50 - \left(\frac{50}{4} + \frac{146}{4}\right) = 1\, gm$

20. 6

$P = P_0 + \frac{mg}{A} = 2\times10^5 N/m^2$

$P'V'^{\gamma} = PV^{\gamma}$

$P' = P\left(\frac{V}{V'}\right)^{\gamma}$

$P' = P\left(1 + \frac{\gamma Ax}{V}\right)$

Also, $P'A + kx - mg - P_0A = ma$ $\Rightarrow \left(\frac{P\gamma A}{V} + k\right)x = ma$

$\Rightarrow \omega = \sqrt{\frac{\left(\frac{P\gamma A^2}{V} + k\right)}{m}} = 6\, rad/s$

## CHEMISTRY

21.

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -RT\ln K; \ H_2O_{(l)} \rightleftharpoons H_2O_{(v)}$

$\ln K = \ln P_{H_2O} = -\frac{\Delta H^\circ - T\Delta S^\circ}{RT} = -\frac{40\times1000 - 300\times120}{8.314\times300} = -1.6$

22. $\Delta E = \frac{hC}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{450\times10^{-9}} = 44\times10^{-20} J$

23. $Mn = (\frac{1}{8}\times4) + 1 = 1.5; \ Si = (\frac{1}{8}\times4) = 0.5; \ Mn_{1.5}Si_{0.5}$

24. Given cell consists of half cells of metal – metal insoluble salt. Each can be reduced to metal – metal ion half cell

25. $NCl_3 + 3H_2O \rightarrow NH_3 + 3HOCl$

$\therefore$ Oxidising

26. Conceptual

27. $I : Co^{+3}$ With $H_2O$ from low spin.

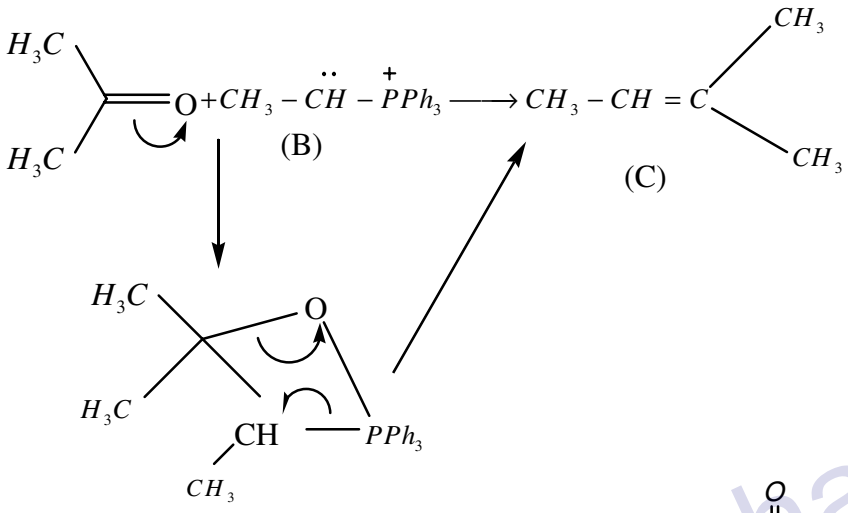
$III : Pt^{+4} : '5d'$ element with all ligands from low spin.

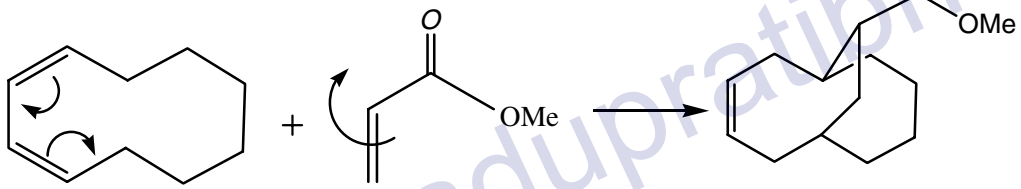
28. $C_2H_5Cl \xrightarrow[DMSO]{KCN} C_2H_5CN \xrightarrow[prolonged\ heat]{H_3O^+} C_2H_5-COOH$

(x) (y)

29. $CH_3CH_2-\overset{+}{P}Ph_3Br^- + C_2H_5ONa \longrightarrow CH_3-\overset{\bullet\bullet}{C}H-\overset{+}{P}Ph_3$

(A) (B)



30. 

31. A) Let number of of moles of B vaporized be a.
Number of moles of B in vapour and liquid phase will be a and (1 – a)
Number of moles of A in vapour and liquid phase will be (1 – a) and a

In liquid phase: $P_T = P_A^o X_A + P_B^o X_B;\ P_A^o a + P_B^o(1-a)$

In Vapour phase: $\frac{1}{P_T} = \frac{Y_A}{P_A^o} + \frac{Y_B}{P_B^o};\ \frac{1}{P_T} = \frac{(1-a)}{P_A^o} + \frac{a}{P_B^o}$

B) A solution of HCl in water forms non-ideal solution with negative deviation. Hence it forms higher boiling azeotrope. Azeotropic mixtures are constant boiling mixtures.

C) $\Delta T_f = K_f \times molaltiy \times i;\ 0.186 = 1.86 \times 0.1 \times i;\ i = 1 \Rightarrow$ It retains its molecular identity.

D) Components of ideal solutions can be separated by distillation method since they form zeotropic mixtures

32. Being amphoteric $SnO_2$ reacts both with HC*l* and NaOH.

$SnO_2$ reacts with HC*l* to form $SnCl_4$. Adsorption of $Sn^{4+}$ results formation of a positively charged colloidal sol.

$SnO_2$ reacts with NaOH to form $Na_2SnO_3$ . Adsorption of $SnO_3^{2-}$ results formation of a negatively charged colloidal sol.

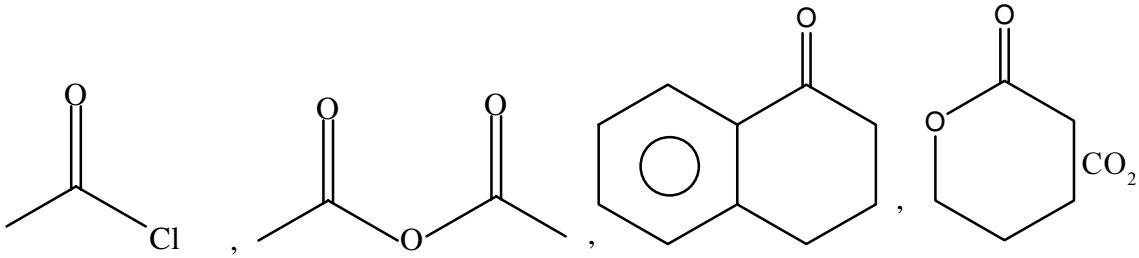
33. HCl, HBr, HI are mono basic

36. Let order with respect to $NO$ be "$x$" and that of $H_2$ be "$y$"

$r = \left(-\frac{dp}{dt}\right)$; $\frac{1.5}{0.25} = \left(\frac{360}{150}\right)^x \Rightarrow x = 2$; $\frac{1.6}{0.8} = \left(\frac{300}{150}\right)^y \Rightarrow y = 1$. Over all order of reaction = 3

37. $\left[Pt(gly)_3\right]^+$ exhibit two geometrical isomers, cis and trans both are optically active.

38. $NF_3$ $SF_6$ $CCl_4$




40. 2

## MATHEMATICS

41. $f(x) = \frac{1}{3^{\{x\}} - 9^{\{x\}} + 1}$

$= \frac{1}{t - t^2 + 1} = \frac{1}{f(t)}$ *where* $3^{\{x\}} = t$

$0 \le \{x\} < 1 \quad \Rightarrow 1 \le 3^{\{x\}} < 3 \Rightarrow 1 \le t < 3$

$f(t) = t - t^2 + 1 \Rightarrow f'(t) = 1 - 2t$

$f'(t) < 0$ *for* $t \in [1, 3]$ *and* $f(t)$ *is decreasing*

$\therefore \max = f(1) = 1$

$f(t) > f(3) = 3 - 9 + 1 = -5$

$-5 < f(t) \le 1 \quad \Rightarrow$ *Range of* $\frac{1}{f(t)} = \left(-\infty, -1/5\right) \cup [1, \infty)$

42. $\frac{ar_1 + br_1}{2} = \Delta = rs \Rightarrow (a+b)r_1 = (a+b+c)r$

$\Rightarrow \frac{r_1}{r} = 1 + \frac{c}{a+b} \Rightarrow 1 < \frac{r_1}{r} < 2.$

43. Consider $\int_0^1 (\alpha - x)^2 f(x)dx = \int_0^1 \left(\alpha^2 f(x) - 2\alpha x f(x) + x^2 f(x)\right)dx$

$= \alpha^2 - 2\alpha^2 + \alpha^2 = 0$

However f(x) assumes only positive values i.e. in (0,1)

$\Rightarrow (\alpha - x)^2 (f(x)) > 0 \Rightarrow$ Integral can't be zero.

44. $\text{Required probability} = \dfrac{\text{Number of favourable cases}}{\text{Total cases}} = \dfrac{1}{9}$

45. Given $|\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OC}| = |\overrightarrow{OD}| = r(say)$

Squaring the given equations, we get

$(x^2+y^2)r^2 + 2xy\overrightarrow{OA}.\overrightarrow{OB} = (x^2+y^2)r^2 + 2xy\overrightarrow{OB}.\overrightarrow{OC}$

$= (x^2+y^2)r^2 + 2xy\overrightarrow{OC}.\overrightarrow{OA} = (x^2+y^2)r^2 + 2xy\overrightarrow{OD}.\overrightarrow{OA}$

$\Rightarrow \overrightarrow{OA}.\overrightarrow{OB} = \overrightarrow{OB}.\overrightarrow{OC} = \overrightarrow{OC}.\overrightarrow{OA} = \overrightarrow{OD}.\overrightarrow{OA}$

$\Rightarrow \cos(\angle AOB) = \cos(\angle BOC)$

$= \cos(\angle COD) = \cos(\angle DOA)$

Since sum of these four angles is $2\pi$ and all angles are equal, ABCD is a square.

46. Let the expression is of the form $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$.

It can be a perfect square if either both expressions are simultaneously perfect squares or both roots common. i.e., $(k-2)^2 + 8k = 0$ and $k^2 - 4(2k-4) = 0$ or $\dfrac{2-k}{k} = \dfrac{-2k}{2k-4}$

Solving we get k can take only one value.

47. $\displaystyle\int \frac{(4\sin x - 1)\cos x}{3 + \sin^2 x - 4\sin x}\,dx$

Put sinx = t $= \displaystyle\int \frac{(4t-1)}{t^2 - 4t + 3}\,dt$

$= \displaystyle\int \frac{(4t-1)}{(t-1)(t-3)}\,dt = \int\left(\frac{a}{t-1} + \frac{b}{t-3}\right)dt$

Then $a = \left[\dfrac{4t-1}{t-3}\right]_{t=1} = -\dfrac{3}{2}$ and $b\left[\dfrac{4t-1}{t-1}\right]_{t=3} = \dfrac{11}{2}$

48. $2012 = 2^2 \times 503$ where 503 is prime

$\Rightarrow$ {503, 1006,2,4,6.....,2012} are not relatively to 2012

Required answer = 2012 – 1008 = 1004

49. The required ratio is $\dfrac{|\overrightarrow{OA}\times\overrightarrow{OB}| + |\overrightarrow{OB}\times\overrightarrow{OC}| + |\overrightarrow{OC}\times\overrightarrow{OA}|}{|\overrightarrow{OC}\times\overrightarrow{OA}|}$

Also, $(\overrightarrow{OA}\times\overrightarrow{OB}) + 3\overrightarrow{OC}\times\overrightarrow{OB} = \overline{0}$

$|\overrightarrow{OA}\times\overrightarrow{OB}| = 3|\overrightarrow{OC}\times\overrightarrow{OB}|$

Similarly $|\overrightarrow{OA}\times\overrightarrow{OC}| = 2|\overrightarrow{OB}\times\overrightarrow{OC}|$

$\Rightarrow \dfrac{|\overrightarrow{OA}\times\overrightarrow{OB}|}{3} = |\overrightarrow{OB}\times\overrightarrow{OC}| = \dfrac{|\overrightarrow{OC}\times\overrightarrow{OA}|}{2} = \lambda$

Therefore required ratio $\dfrac{6\lambda}{2\lambda} = 3$.

50. Let us form that equation having roots $y = g(x_i)$ i.e., $y = x^2 - 2$

$x = \sqrt{y+2}$

$\Rightarrow \left(\sqrt{y+2}\right)^5 + \left(\sqrt{y+2}\right)^2 + 1 = 0$

$\Rightarrow y^5 + 10y^4 + 40y^3 + 79y^2 + 74y + 23 = 0$

$\therefore g(x_1).....g(x_5) =$ Product of roots

= -23.

51. We have $f(x) = \begin{cases} (x+1)(2x-1) \text{ for } x < -1 \\ -(x+1)(2x-1) \text{ for } -1 \le x < 0 \\ (x+1) \text{ for } 0 \le x < 1 \\ (x+1)(2x-1) \text{ for } x \ge 1 \end{cases}$

$Lf'(-1) = -3$ and $Rf'(-1) = 3 \Rightarrow f'(-1)$ does not exist

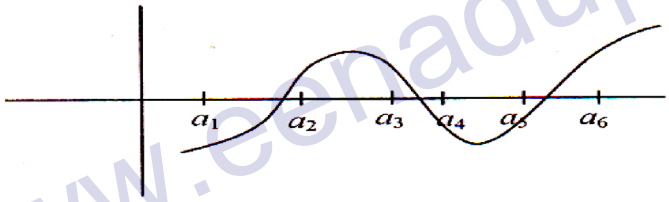
At $x = -1$, LHL = RHL = 0 and f(-1) = 0

$\therefore$ f is contiuous at $x = -1$

$Lf'(0) = -1$ and $Rf'(0) = 1 \Rightarrow f'(0)$ does not exist

$Rf'(1) = \left[(x+1)2 + (2x-1)(1) \text{ at } x = 1\right] = 5$

52. $= (x-a_1)(x-a_3)(x-a_5) + (x-a_2)(x-a_4)(x-a_6)$




53. $f(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$

Suppose for a natural number k, f(x) assumes values $\sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$, then the same value is assumed again by the function at $P = n\pi + (-1)^n\left(\frac{\pi}{4} + k\right)$

Irrespective of 'n' P can never be a natural number.

Hence f(x) is one-one.

Further f(x) cannot attain all values in $\left[-\sqrt{2}, \sqrt{2}\right]$ which $\sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$ attains irrational values.

e.g. f(x) = 0 when $\frac{\pi}{4} + x = n\pi$ which is not true for $x \in N$. Hence into.

54. A.M. of distances of focus from two directrices is CP i.e. 10

CP/distance between two directrices = $\frac{(eccentricity)^2}{2}$

$\Rightarrow \frac{10}{4} = \frac{e^2}{2} \Rightarrow e = \sqrt{5}$

55. Probability that exactly 2 persons will get nothing

$$= {}^{n+2}C_2 \frac{n!}{(n+2)^n} = \frac{(n+2)(n+1)n!}{2.(n+2)^n} = \frac{(n+1)!}{2(n+2)^{n-1}}$$

Probability that exactly 3 persons will get nothing $= {}^{n+2}C_3 . \frac{{}^{n-1}C_1 . {}^{n}C_2 . (n-2)!}{(n+2)^n}$

56. $f(x) = (x-a)(x-c)(x-e) + \lambda(x-b)(x-d)$

$\Rightarrow f(a) = \lambda(a-b)(a-d)$

$\Rightarrow f(b) = (b-a)(b-c)(b-e) < 0$

$f(c) = \lambda(c-b)(c-d)$

$f(d) = (d-a)(d-c)(d-e) > 0$

$f(e) = \lambda(e-b)(e-d)$

If $\lambda > 0 f(a) > 0$, a root lies between b and a.

If $\lambda < 0 f(e) < 0$, a root lies between e and d.

Always a root lies between d and $b \Rightarrow$ all roots are real and distinct as exactly two can't be real.

If $\lambda = 0$ roots are a,c and e.

57. $|z_1 + z_2 + ..... + z_n|^2 = 0$

$\Rightarrow |z_1|^2 + |z_2|^2 + ..... + |z_n|^2 + z_1\overline{z}_2 + z_2\overline{z}_1 + ..... = 0 \qquad \rightarrow (1)$

Also $P = \sum_{j=1}^{n}\sum_{i=1}^{n}\left(\frac{z_i}{z_j}\right)$

$= n + \frac{z_1\overline{z}_2}{|z_2|^2} + \frac{z_2\overline{z}_1}{|z_1|^2} + ....$

$P = n + \frac{(z_1\overline{z}_2 + z_2\overline{z}_1 + .....)}{r^2}$, where $|z_1| = |z_2| = ..... = |z_n|$

$\Rightarrow \text{Re}(P) = n + \frac{\text{Re}(z_1\overline{z}_2 + z_2\overline{z}_1 + .....)}{r^2}$

$= n + \frac{(-nr^2)}{r^2} = 0$. (Using (1))

Or

$z_1, z_2, z_3, .....z_n$ can be taken as $n^{th}$ roots of unity

58. m = number of distinct terms in $\left[\sum x^2 + \sum \frac{1}{x^2y^2} + 2\sum xy + 2\sum \frac{1}{xy^2z} + 4\sum \frac{1}{x} + 2\sum \frac{x}{yz}\right]$=18

n = number of distinct terms in $\left[\sum x^2 + \sum \frac{1}{x^2} + 2\sum xy + 2\sum \frac{1}{xy} + 2\sum (1) + 2\sum x\left(\frac{1}{y} + \frac{1}{z}\right)\right]$=19

$\therefore |m - n| = 1$

59. $\det\left(A^2 - I_3\right) = \det\left(A^2 - AA^T\right) = \det\left(A\left(A - A^T\right)\right)$

$= \det\left(A - A^T\right)\det\left(A\right)$

Further $\det\left(A\right) = \pm 1$, and matrix $A - A^T$ is a skew symmetric matrix with odd order hence its determinant is 0.

60. Number $= 2^6\ 3^4\ 5^2$

$\alpha$ = divisors of (4k + 1) form $(k \geq 1)$

= number of combinations of any number of elements from each of $\{9, 81\}$ and $\{5, 25\}$

= (2 + 1) ( 2 + 1) – 1 = 8

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