JEE-ADVANCED

MODEL GRAND TEST PAPER

Time : 3 hrs] [Number of questions : 60

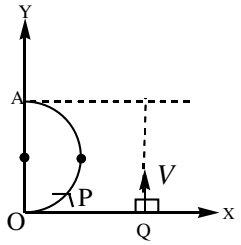
PHYSICS

SECTION I

Single Correct Answer Type

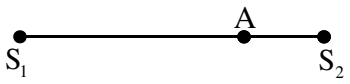
This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. A particle P starts from origin as shown and moves along a semicircular path. Another particle Q crosses $x-$axis at the instant particle P leaves the origin. Q moves with constant speed V parallel to $y-$axis and is all the time having $y-$co-ordinate same as that of P. When P reaches diametrically opposite point A, its average speed is :



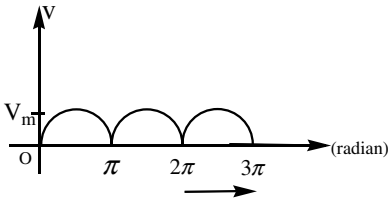
(A) πV (B) $\frac{\pi V}{2}$ (C) $\frac{V}{2}$ (D) V

2. If $y_1 = 5(\text{mm})\sin \pi t$ is the equation of oscillation of source S_1 and $y_2 = 5(\text{mm})\sin(\pi t + \pi/6)$ be that of S_2 and it takes 1 s and 1/2 s for the transverse waves to reach point A from sources S_1 and S_2 respectively then the resulting amplitude at point A, is :



(A) $5\sqrt{2+\sqrt{3}}$ mm (B) $5\sqrt{3}$ mm (C) 5 mm (D) $5\sqrt{2}$ mm

3. The average and effective values for the wave shape (positive half cycles of sine curve) shown in the figure are :



(A) $\frac{2}{\pi}V_m$ and $\frac{V_m}{2}$ (B) $\frac{V_m}{\pi}$ and $\frac{V_m}{\sqrt{2}}$ (C) $\frac{2}{\pi}V_m$ and $\frac{V_m}{\sqrt{2}}$ (D) $\frac{V_m}{\pi\sqrt{2}}$ and $\frac{V_m}{\sqrt{2}}$

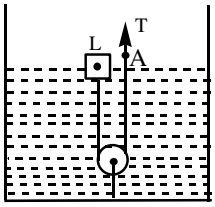
4. A smooth tunnel is dug along the radius of the earth that ends at the centre of earth. A ball is released from the surface of earth along the tunnel. If the coefficient of restitution is 0.2 between the end of tunnel and ball then the total distance travelled by the ball before second collision at the centre of earth is

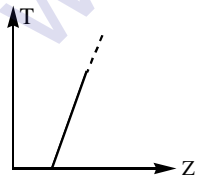
(A) $\frac{6}{5}R$ (B) $\frac{7}{5}R$ (C) $\frac{9}{5}R$ (D) $\frac{3}{2}R$

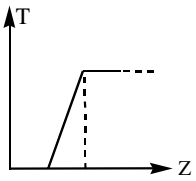
5. Two particles execute SHM along x-axis about the origin with same amplitude 'a' and frequency ω. At a certain instant, they are found at a distance a/3 from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two? [Assume that the particles do not collide]

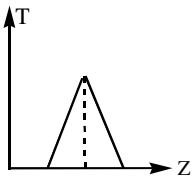
(A) $\cos^{-1}\frac{7}{9}$ (B) $\cos^{-1}\frac{5}{9}$ (C) $\cos^{-1}\frac{4}{9}$ (D) $\cos^{-1}\frac{1}{9}$

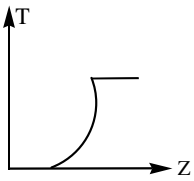
6. A cubical wooden block of side L and relative density 0.5 is floating on the surface of water as shown in the figure. A string is attached to it through an ideal pulley fixed to the bottom of vessel as shown in figure. The graph of tension T in the string *vs* distance of the bottom of the block from the free surface of the water when the end A of the rope is slowly pulled up will be (block is moving very slowly and the vessel is very wider):



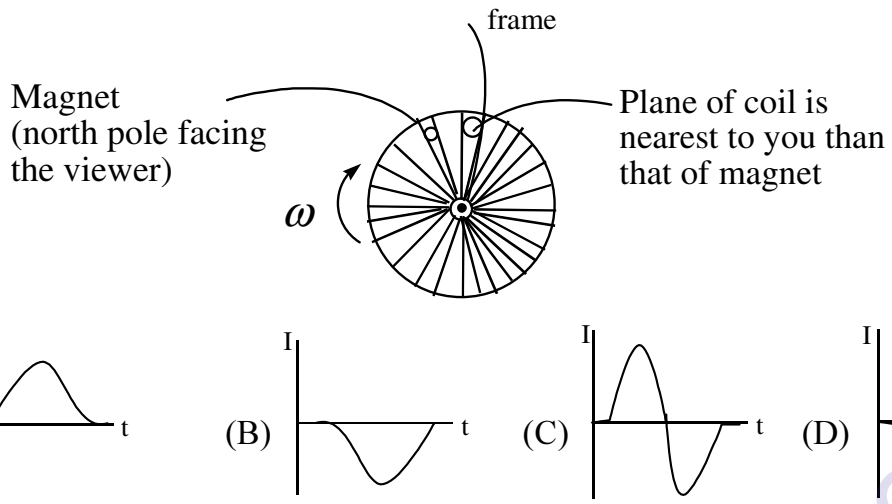
(A) 

(B) 

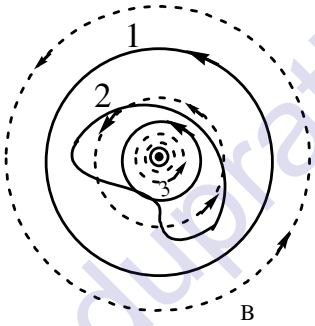
(C) 

(D) 

7. In a bicycle speedometer , a bar magnet is attached to the spokes of the wheel and a coil is attached to the frame so that the north pole of the magnet moves past it once for every rotation of the wheel. As the magnet moves past the coil, a pulse of current is induced in the coil. A computer then measures the time between consecutive pulses and computes the bicycle's speed. Figure shows the magnet about to move past the coil. Which of the graphs best represents the resulting current pulse? (Take counter clockwise current in figure to be positive):



8. Consider three closed loops drawn using solid line in the magnetic field (magnetic field lines are drawn using dotted line) of an infinite current-carrying wire normal to the plane of paper as shown.

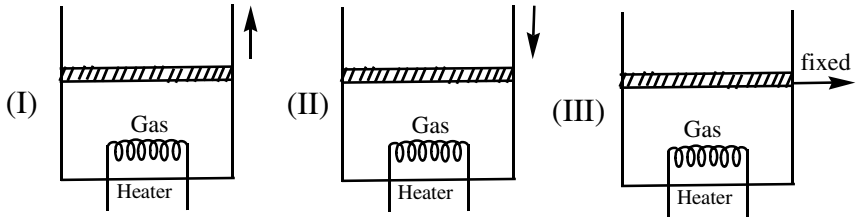


If a,b and c represent the values of line integrals of the magnetic field along the paths 1,2 and 3 respectively, then :

(A) $a > b > c$ (B) $a = c > b$

(C) $a = b = c$ (D) $c > b > a$

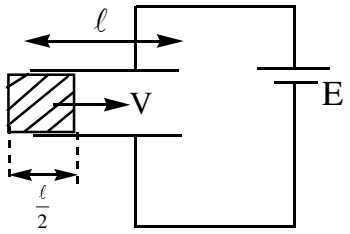
9. A sample of gas is heated by three different methods from same initial state as shown. In each method, heat supplied is same. In method I piston moves up by some amount. In method II piston moves down and in method III piston does not move. Specific heat of the gas calculated in each of the methods to be C_I, C_{II} and C_{III} respectively. If the piston, walls of vessel are made of insulators and piston is friction less, then

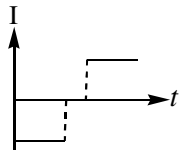


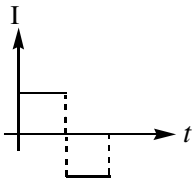
(A) $C_I > C_{II} > C_{III}$ (B) $C_{II} > C_I > C_{III}$

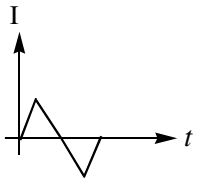
(C) $C_{III} > C_I > C_{II}$ (D) $C_I > C_{III} > C_{II}$

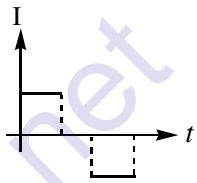
10. A dielectric slab of area A and thickness d is inserted between the plates of capacitor of area 2 A and distance between the plates d, with a constant speed V as shown in figure. The capacitor is connected to a battery of emf $\mathcal{E}$. The current in the circuit varies with time as: (Current through the cell in the direction of emf is positive)



(A) 

(B) 

(C) 

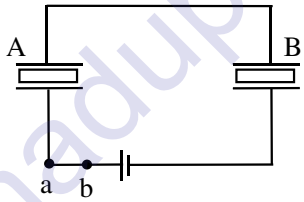
(D) 

SECTION II

Multiple Correct Answer(s) Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

11. Identical dielectric slabs are inserted into two identical capacitors A and B. These capactors and a chargeable battery are connected as shown in the figure. Now the slab of capacitor B is pulled out with battery remaining connected:



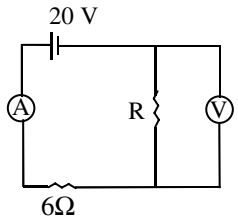
(A)During the process, positive charge flows from *a to b*

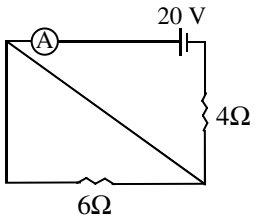
(B) Finally, charge on capacitor B will be less than that on A

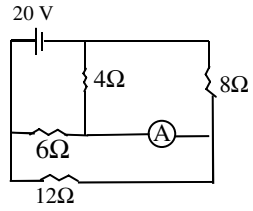
(C) During the process, positive work is done by the external force F, part of which appears as heat in the circuit.

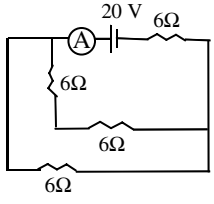
(D) During the process, internal energy of the battery increases.

12. The ammeters connected in the following circuits have zero resistance. The voltmeter in figure (A) has infinite resistance and shows a reading 8V. The value of resistance R has not been specified. Which of the following circuit (s) has same current in the ammeter?

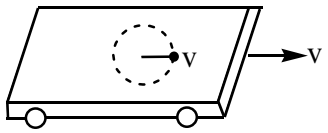
(A) 

(B) 

(C) 

(D) 

13. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of the train, with same constant speed v relative to train. Mark the correct option (s).



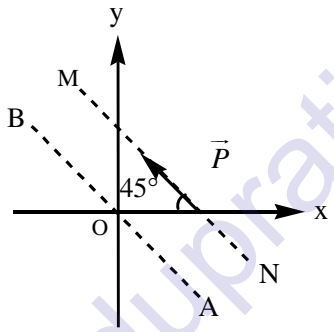
(A) Maximum speed of the bob is 2v in ground frame.

(B) Tension in string connecting the bob is $\frac{4mv^2}{\ell}$ at an instant.

(C) Tension in string is $\frac{mv^2}{\ell}$ at all the moments.

(D) Minimum speed of bob is zero in ground frame.

14. A particle is moving with constant momentum $\vec{P}$ along line MN as shown in figure. Line AB is parallel to MN. Mark the correct statements.



(A) Angular momentum of particle about any point on line MN is zero.

(B) With respect to any reference point on line AB, angular momentum vector of the particle is constant.

(C) With respect to any reference point in xy - plane in third quadrant, angular momentum vector of the particle is in positive z -direction.

(D) With respect to any reference point in xy - plane in third quadrant angular momentum vector of the particle is in negative z -direction.

15. In a photoelectric effect experiment, if f is the frequency of radiation incident on the metal surface and I is the intensity of the incident radiations, then mark the correct statement(s).

(A) If f is increased keeping I and work function constant then stopping potential and maximum kinetic energy of photoelectron increase.

(B) If distance between cathode and anode is only changed then stopping potential remains same.

(C) If I is increased keeping f and work function constant then stopping potential remains same and saturation current increases.

(D) Work function is decreased keeping f and I constant then stopping potential and maximum kinetic energy of photoelectrons increase.

SECTION II

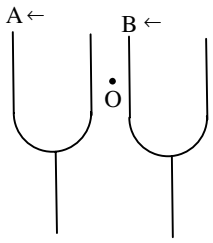
Multiple Correct Answer(s) Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

16. The principal section of glass prism is an isosceles ΔPQR with $PQ = PR$. The face PR is silvered. A ray is incident perpendicularly on face PQ and after two reflections, it emerges from base QR, normal to it. The angle of the prism is given by $\frac{\pi}{\alpha}$ rad. Find the value of α.

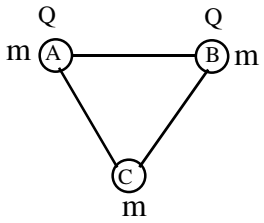
17. A balloon is filled with helium at the atmospheric pressure P_0. The volume of the balloon is V_0. The balloon is made of the material of density ρ and its mass is m. After being released, the balloon bursts at an altitude where the atmospheric pressure is $(P_0/2)$. Just before bursting, the balloon has a volume of $1.25\,V_0$. If the maximum stress that the balloon material can withstand is given by $\frac{750}{k}P_0$, find the value of k. Assume that the temperature of helium remains constant, the balloon remains spherical, thickness of balloon material is very small when compared with radius and the density of the material remains virtually constant. [Take : $V_0 = 8m^3, \rho = 1500kg/m^3, m = 18kg$]

18. Two tuning forks A and B each of natural frequency 85 Hz move with velocity 10m/s relative to stationary observer 'O'. Fork A moves away from the observer while the fork B moves towards him as shown in the figure. A wind is blowing with a speed 10 m/s in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s]



19. The internal energy of a monoatomic ideal gas is $1.5\,nRT$. One mole of helium is kept in a cylinder of cross-section $8.5\,cm^2$. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. The temperature rises through 2^0C. The distance moved by the piston is given as $(\alpha \times 10^{\beta})$m in scientific notation. Find the value of $\alpha + \beta$. [Take $R = \frac{25}{3}$ in SI units, atmospheric pressure=100kPa]

20. Three small balls of equal mass m are connected by light insulating inextensible threads of length ℓ each and kept on a level smooth non-conducting ground. The balls A and B are given charge Q each. The strings are all taut. The string connecting A and B suddenly snaps. What is the maximum speed (in m/s) of C during the resulting motion? $Q = 1\mu C, \ell = 1.5m$, mass m=1g.



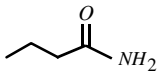
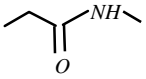
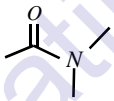
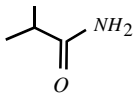
CHEMISTRY

SECTION I

Single Correct Answer Type

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

21. Which of the isomeric compounds would you expect to have the lowest boiling point

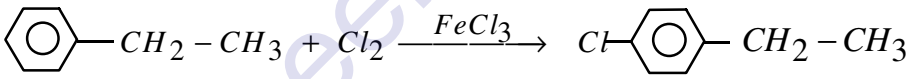
(A)  (B)  (C)  (D) 

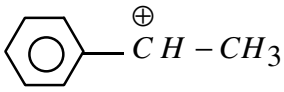
22. $X + Mg \xrightarrow[ether]{dry} Y \xrightarrow[2.H_3O^+]{1.Z} CH_3 - CH_2 - CH_2 - OH$

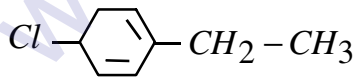
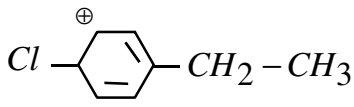
Identify X and Z in the above sequence of reaction

(A) CH_3CH_2Br and $HCHO$ (B) CH_3Br and $CH_2 - \underset{\diagup\ \diagdown}{O} - CH - CH_3$

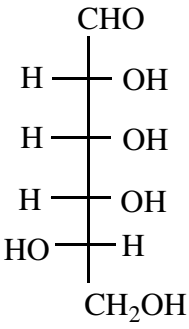
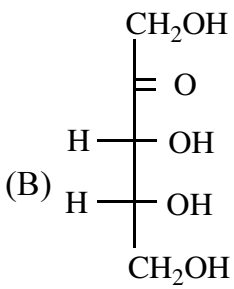
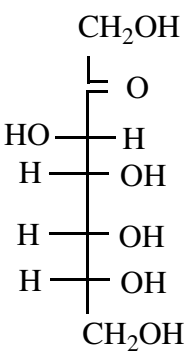
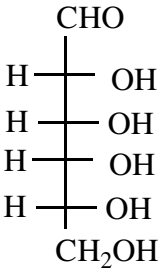
(C) CH_3Br and CH_3CH_2CHO (D) CH_3Br and CH_3CHO

23.  Which of the following is a key intermediate in the reaction shown above.

(A)  (B) 

(C)  (D) 

24. A pyranose form is a cyclic hemiacetal with a six-membered ring. Which of the following compounds can not exists in pyranose form

(A)  (B)  (C)  (D) 

25. The equation of state of a gas is $(P + aT^2/V)V^C = (RT + b)$ where a, b, c and R are constants. The isotherms can be represented by $P = AV^m - BV^n$ where A and B depends only on temperature and

(A) $m = -c$ and $n = -1$ (B) $m = c$ and $n = 1$ (C) $m = -c$ and (D) $m = c$ and $n = -1$

26. A sample of contains one Fe(III) for every three Fe(II). Then the value of x will be

(A) 8/9 (B) 9/8 (C) 4/3 (D) 6/5

27. Liquid A undergoes self ionization to form B^+ and C^- with the equilibrium $2A \rightleftharpoons B^+ + C^-$ for the equilibrium is 10^{-30}. How many C^- ions are present per cc of the liquid A (assuming A behaves like a base).

(A) 10^{-8} (B) 10^{-15} (C) 6.023×10^5 (D) 12.04×10^5

28. In the following sub questions choose the correct answer from among the following possibilities and select correct code of your answers (Answer of P,Q R and S respectively):

(P) The most stable low valent halide (1) GeX_2 (2) SnX_2 (3) PbX_2

(Q) A non existing halide (1) $SnCl_4$ (2) $PbCl_4$ (3) PbI_4

(R) A purely acidic oxide (1) PbO_2 (2) SnO_2 (3) SiO_2

(S) Thermally most stable hydride (1) NH_3 (2) PH_3 (3) AsH_3

(A) 3,2,1,3 (B) 1,3,3,1 (C) 3,3,3,1 (D) 1,1,1,3

29. Assume that $\left[Fe(H_2O)_6\right]^{+2}$ ion is a high spin complex and the enthalpy of hydration of Fe^{+2} ion is -12.4kcal/mole which is higher than that expected in the absence of crystal field stabilization energy then the estimated value of Δ_0 for this ion is

(A) $7234cm^{-1}$ (B) $8680cm^{-1}$ (C) $1808.4cm^{-1}$ (D) $10850cm^{-1}$

30. Given the data at 25^0C

$Ag^+ + e^- \rightarrow Ag$; $E^0 = +0.800V$

$AgI + e^- \rightarrow Ag + I^-$; $E^0 = -0.152V$

then the value of $\log K_{sp}$ for AgI is

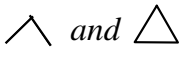
(A) -16.13 (B) +8.612 (C) -37.83 (D) -8.12

SECTION II

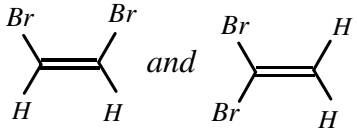
Multiple Correct Answer(s) Type

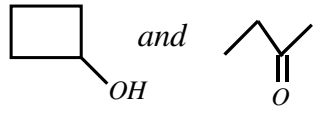
This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

31. Which of the following represents a pair of constitutional isomers?

(A)  *and*

(B) $CH_3 - CH_2 - CH = CH_2$ *and* $CH_3 - CH = CH - CH_3$

(C) 

(D) 

32. Some statements are given about following compound

$$CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH(OCH_3)_2$$

Identify correct statements

(A) It gives yellow precipitate with I_2 in aq $NaOH$ Solution

(B) It does't form silver mirror with Tollens reagent

(C) One of Hydrolysis products of given compound gives red precipitate with Benedicts solution

(D) Both, the given compound and its one of hydrolysis products gives positive test with 2, 4 – DNP

33. Which is incorrectly matched for refining of metals?

(A) Si, Ge from impurities $\Rightarrow$ zone refining method

(B) Copper refined from its metaloxide $\Rightarrow$ cupellation method

(C) Al from impurities $\Rightarrow$ Hoop's method

(D) Zn, Cd from impurities with high boiling $\Rightarrow$ point liquation

34. In the aqueous solution of soaps above CMC :

(A) the cations associate to form the aggregates and not directed towards water.

(B) the anions associate to form the clusters of colloidal dimension

(C) the polar ends of the ions forming the clusters are directed towards water

(D) the non-polar (hydrocarbon) ends are directed towards water

35. Which of the following reactions give the same nitrogen containing gaseous product?

(A) Heating of NH_4NO_2

(B) By passing NH_3 over heated CuO

(C) Heating of $Ba(N_3)_2$

(D) $Cu+$ cold and dilute HNO_3

SECTION III

Integer Answer Type

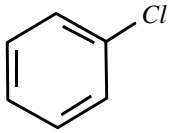
This section contains 5 questions. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

36. The enthalpy of monoatomic gas at 27^0C is $300y$ calorie then the value of 'y' is

37. How many of the following would be expected to give a positive test with Benedict's reagent?

D-Galactose	D-Fructose	Sucrose	D-ribose
Lactose	Gluconic acid	Maltose	

38. Number of chlorine atoms present in a molecule of organic product of following reaction



$$+\quad CCl_3-\overset{\overset{O}{\|}}{C}-H \xrightarrow{H_2SO_4}$$

Chlorobenzene **Trichloroacetaldehyde**

39. The no of the possible orbitals (upto n = 3) which fulfill all the following conditions

Case – I → having only one radial node

Case – II → having XY plane as angular node

Case – III → having only two maxima if a curve is plotted between radial distribution function vs radial distance

40. A certain reaction B^{n+} is getting converted to $B^{(n+4)+}$ in solution. The rate constant of this reaction is measured by titrating a known volume of the solution with a reducing agent which reacts only with B^{n+} and $B^{(n+4)+}$. In this process it converts B^{n+} to $B^{(n-2)+}$ and $B^{(n+4)+}$ to $B^{(n-1)+}$. At, $t = 0$, the volume of reagent consumed is 25 ml and at t = 10 minute, the volume used is 32 ml. If the rate constant for the conversion of B^{n+} of $B^{(n+4)+}$ is $x \times 10^{-2}\ \text{min}^{-1}$, then the value of '$x$' is (Near by Integer)

MATHEMATICS

SECTION I

Single Correct Answer Type

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

41. If $\sum_{r=1}^{n} t_r = \frac{n}{8}(n+1)(n+2)(n+3)$ then $\sum_{r=1}^{n} \frac{1}{t_r} =$

(A) $\frac{n(n+1)}{(n+2)(n+3)}$ (B) $\frac{n(n+1)}{2(n+2)(n+3)}$ (C) $\frac{n(n+1)}{(n+1)(n+2)}$ (D) $\frac{n(n+3)}{2(n+1)(n+2)}$

42. $\sum_{p=1}^{32}(3p+2)\left\{\sum_{r=1}^{10}\left(\sin\frac{2r\pi}{11} - i\cos\frac{2r\pi}{11}\right)\right\}^{p} =$

(A) $48i$ (B) $16(1-i)$ (C) $48(1-i)$ (D) $16(i-1)$

43. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$ then $\sum_{0 \le i < j \le n} \sum (Ci + Cj)^2 =$

(A) $(n-1)2n_{c_n} + 2^{2n}$ (B) $n\left(2n_{c_n}\right) + 2^{2n}$

(C) $(n+1)2n_{c_n} + 2^{n}$ (D) $n\left(2n_{c_n}\right) + 2^{n}$

44. If two small squares of selected at random from a chess board then the probability that they have exactly one common vertex is

(A) $\frac{1}{144}$ (B) $\frac{7}{72}$ (C) $\frac{1}{72}$ (D) $\frac{7}{144}$

45. $\lim_{n\to\infty}\prod_{r=3}^{n}\left(\frac{r^3-8}{r^3+8}\right)=$

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$

46. The largest term of the sequence $a_n=\frac{n^2}{n^3+200}$ is

(A) a_6 (B) a_7 (C) a_8 (D) a_9

47. The range of values of α for which the line $2y=gx+\alpha$ is a normal to the circle $x^2+y^2+2gx+2gy-2=0$ for real values of g is

(A) $[1,\infty)$ (B) $[-1,\infty)$ (C) $(0,1)$ (D) $(-\infty,1]$

48. $(x-1)(y-2)=5$ and $(x-1)^2+(y+2)^2=r^2$ intersect in the points A,B,C,D and the centroid of the triangle ABC lies on the line $y=3x-4$. Then the locus of D is

(A) $y=3x$ (B) $x^2+y^2+3x+1=0$

(C) $3y=x+1$ (D) $y=3x+1$

49. The position vectors of the point A,B,C are respectively $\hat{i}-\hat{j}-3\hat{k},\ 2\hat{i}+\hat{j}-2\hat{k}$ *and* $-5\hat{i}+2\hat{j}-6\hat{k}$. If the internal angular bisector of $\underline{\lfloor A}$ in ΔABC meet BC at D then the length of AD is

(A) $\frac{1}{4}$ (B) $\frac{11}{2}$ (C) $\frac{15}{2}$ (D) $\frac{3\sqrt{10}}{4}$

50. The number of roots of the equation $\cos^7 x+\sin^4 x=1$ in the interval $(-\pi,\pi)$ is

(A) 4 (B) 3 (C) 2 (D) 1

SECTION II

Multiple Correct Answer(s) Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

51. $\int\frac{dx}{(1+x)\sqrt{1+x-x^2}}=$

(A) $\cos^{-1}\frac{2\sqrt{1+x-x^2}}{\sqrt{5}(x+1)}+C$ (B) $2\tan^{-1}\left(\frac{\sqrt{1+x-x^2}+1}{2}\right)+C$

(C) $\sin^{-1}\left(\frac{3x+1}{5(x+1)}\right)+C$ (D) $\sin^{-1}\left(\frac{3x+1}{\sqrt{5}(x+1)}\right)+C$

52. The tangent at a point P on a curve meets X-axis in A. If the area of the triangle OAP (O is origin) is a^2 (a constant), then the equation of the curve is

(A) $x = cy + \frac{a^2}{y}$ (B) $y = cx + \frac{a^2}{x}$ (C) $x = cy - \frac{a^2}{y}$ (D) $y = cx - \frac{a^2}{x}$

53. If the roots of the equation $x^2 - ax - b = 0$ $(a, b \in R)$ have absolute value less than 1, then

(A) $|b| < 1$ (B) $a + b < 1$ (C) $b - a < 1$ (D) $a + b = 0$

54. Let A,B,C be square matrices of same order and I is the unit matrix of same order such that $A + B + C = AB + BC + CA$. Consider the following statements

(i) $ABC = AC - CA$ (ii) $BCA = BA - AB$ (iii) $CAB = CB - BC$

(A) (i) and (ii) are equivalent (B) (ii) and (iii) are equivalent

(C) (i) and (iii) are equivalent (D) (i) , (ii) and (iii) are equivalent

55. Which of the following statement is / are correct

(A)The number of solutions of equation $Tan^{-1}\left(\frac{x}{1-x^2}\right) + Tan^{-1}\left(\frac{1}{x^3}\right) = \frac{3\pi}{4}$ belonging to the interval (0,1) is 2

(B) $Tan^{-1}\left(\frac{1}{2}\right) + Tan^{-1}\left(\frac{1}{8}\right) + Tan^{-1}\left(\frac{1}{18}\right) + Tan^{-1}\left(\frac{1}{32}\right) + \ldots$. to n terms is $\frac{\pi}{4} - Tan^{-1}\left(\frac{1}{2n+1}\right)$

(C) If $0 \le x \le 1$ and $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$, then $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

(D) The value of $Sin^{-1}\left(\sin\frac{4\pi}{3}\right) + Cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is $\frac{\pi}{2}$

SECTION II

Multiple Correct Answer(s) Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

56. The letter of the word MOTHER are arranged in all possible ways and the resulting words are arranged as in DICTIONARY. If the rank of the word is K, then the number of positive integral divisors of K is

57. In the triangle ABC, points K and L are taken on the segments AB and BC such that $AK : KB = 1 : 2$ and $BL : LC = 1 : 2$. Let P be the point of intersection of the lines AL and CK. If the area of ΔBCP is 2 square units then twice the area of triangle ABC is

58. If $I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx,$ then the integral part of I is

59. If the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta, b\sin\theta)$ meets the auxiliary circle in two points A,B such that the chord AB subtends a right angle at the centre, then the eccentricity of the ellipse is $\frac{1}{\sqrt{\alpha+\beta\sin^2\theta}}$ where $(\alpha+\beta)^2 =$

60. In triangle ABC, $\frac{tanA-\tan B}{\tan A+\tan B}=\frac{c-b}{c}$ then $2\tan^2 A =$

***

JEE-ADVANCED

MODEL GRAND TEST PAPER - I

KEY SHEET

Physics:

01) B 02) C 03) C 04) B 05) A 06) B 07) D 08) C 09) D 10) D

11) ACDorAD 12) AD 13) ACD 14) ABC 15) ABCD

16) 5 17) 2 18) 5 19) 1 20) 2

Chemistry:

21) C 22) A 23) D 24) B 25) A 26) A 27) C 28) C 29) D 30) A

31) BCD 32) ABCD 33) BD 34) BC 35)ABC 36) 5 37) 5

38) 5 39) 1 40) 2

Mathematics:

41) D 42) C 43) A 44) D 45) B 46) B 47) B 48) A 49) D 50)B

51) AD 52)AC 53) ABC 54) ABCD 55) BCD 56) 4 57) 7 58) 9

59) 4 60) 6

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MODEL GRAND TEST PAPER - I

HINTS & SOLUTIONS

PHYSICS

1. Average speed $\frac{dist}{time} = \frac{\pi R}{2R/V} = \frac{\pi V}{2}$

2. Phase difference between the waves at A is

$$\phi = \frac{2\pi}{\lambda} x + \phi_0$$

$$= \frac{2\pi}{\lambda} C(\Delta t) + \phi_0$$

$$= \omega(\Delta t) + \phi_0$$

$$= \pi\left[\frac{1}{2}\right] + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi} = 5$$

3. $V_{av} = \frac{V_m}{\pi}\int_0^{\pi} \sin\theta d\theta = \frac{2V_m}{\pi}$

$$V_{eff}^2 = \frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta d\theta = \frac{V_m^2}{2}$$

4. In the tunnel, motion of the particle is simple harmonic with angular frequency $\omega = \sqrt{\frac{g}{R}}$.

Just before collision, $V_0 = A\omega = R\sqrt{\frac{g}{R}} = \sqrt{gR}$

Just after collision, $v = eV_0 = \sqrt{gR}/5$

Let new amplitude be A^1, then

$$A^1 = \frac{v}{\omega} = \frac{\sqrt{gR}/5}{\sqrt{g/R}} = \frac{R}{5}$$

Net distance $= R + (R/5) + (R/5) = \frac{7R}{5}$

5. Let $x_1 = a\sin\omega t$ and $x_2 = a\sin(\omega t + \delta)$ be two S.H.M.s

$\frac{a}{3} = a\sin\omega t$ and $\frac{-a}{3} = a\sin(\omega t + \delta)$

$\sin\omega t = \frac{1}{3}$ and $\sin(\omega t + \delta) = \frac{-1}{3}$

Eliminating t, $\frac{1}{3}\cos\delta + \sqrt{1-\frac{1}{9}}\sin\delta = \frac{-1}{3} \Rightarrow \cos\delta + \sqrt{8}\sqrt{1-\cos^2\delta} = -1$

$\Rightarrow 8(1-\cos^2\delta) = (1+\cos\delta)^2$

$\Rightarrow 8 - 8\cos^2\delta - 1 - \cos^2\delta - 2\cos\delta = 0$

$9\cos^2\delta + 2\cos\delta - 7 = 0$

$\cos\delta = -1$ or 7/9 i.e., $\delta = 180^0$ or $\cos^{-1}(7/9)$

Now $\upsilon_1 = a\omega\cos\omega t$ and $\upsilon_2 = a\omega\cos(\omega t + \delta)$

If we put $\delta = 180^0$

We find that υ_1 and υ_2 are of opposite signs.

Hence $\delta = 180^0$ is not applicable.

$\therefore \quad \delta = \cos^{-1}(7/9)$

6. Let y is the additional length immersed. Distance of bottom face of cube from free surface of liquid is $z = \frac{L}{2} + y \Rightarrow y = z - \frac{L}{2}$

Again, tension = extra force of buoyancy developed

$T = y(L^2)\rho g = \left(z - \frac{L}{2}\right)L^2\rho g$

$T = (L^2\rho g)z - \frac{L^2}{2}\rho g$

$T = k_1 z - k_2$ upto $z = L$

After z = L, buoyancy and hence T does not change as the body is completely immersed.

7. Initially for half revolution when magnet moves towards the loop, flux increases $\Rightarrow$ clockwise current .

When magnet moves away from the loop, flux decreases $\Rightarrow$ anticlockwise current

8. $\oint \vec{B} d\vec{l} = \mu_0 I_{enclosed}$

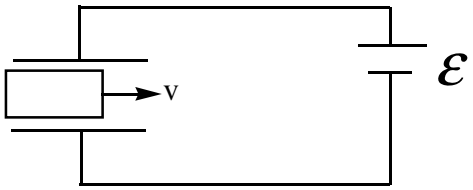
Since $I_{enclosed}$ is same

9. $\because Q = nC\Delta T$

Here Q is same for all the processes bout $W_I = +ve, W_{II} = -ve$ and $W_{III} = 0$ and $\Delta U + W = Q$

$\Rightarrow \Delta U_I < \Delta U_{III} < \Delta U_{II} \Rightarrow \Delta T_I < \Delta T_{II} < \Delta T_{III}$

$\Rightarrow C_I > C_{III} > C_{II}$



Area of dielectric slab = A

Area of plate of capacitor = 2A

After time 't' slab will cover distance 'vt' inside the capacitor and let $A = l \times b$

Now capacitance at time 't'

$$C(t) = \frac{K(vt \times b)\varepsilon_0}{d} + \frac{(l - vt) \times b\varepsilon_0}{d}$$

$$Q(t) = CV = C\varepsilon = C(t) \times \varepsilon$$

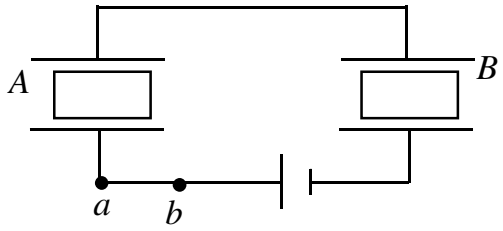
$$I = \frac{dQ(t)}{dt} = \frac{Kv \times b\varepsilon_0}{d} - \frac{vb\varepsilon_0}{d}$$

$$I = \frac{(K-1)vb\varepsilon_0}{d} = \text{constant}$$

i.e. $I = \text{constant}$

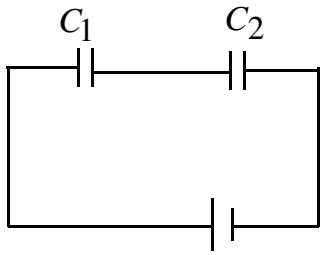
Also when slab totally enters the capacitor after that it goes out from the other side, it will move inside the capacitor for some time with C remaining constant and than C will start decreasing. So current will reverse its direction.

11. As slab is pulled out of capacitor 'B' capacitance of B decreases.



Now consider given circuit

$\because$ As capacitance of B decreases net capacitance also decreases



$\Rightarrow q = CV$ *i.e.* charge on A decreases i.e., +charge moves from a to b but still A and B are in series

$\therefore Q_A = Q_B$

External force 'F' does work while taking slab out. This work (energies) is partly absorbed by battery since it does –ve work and rest appears as heat in circuit since charge is flown.

12. For circuit (A) $I_A = \dfrac{20-V}{6} = \dfrac{20-8}{6} = 2A$

For circuit (B) $I_A = \dfrac{20}{4} = 5A$

For circuit (c) $I_A = 0$

(Balanced Wheatstone's Bridge)

For circuit (d) 6Ω is in parallel with $6+6=12\Omega$

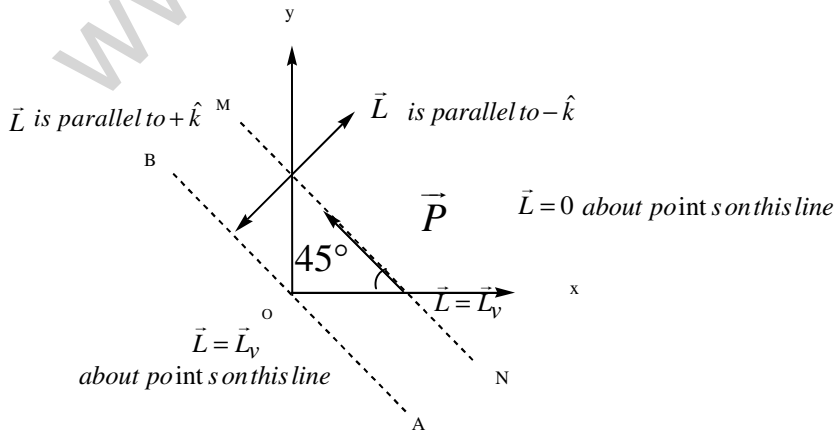
Their effective resistance is $\dfrac{6\times12}{6+12} = 4\Omega$

Net resistance of the circuit $= 4+6 = 10\Omega$

$I_A = 20/10 = 2A$

13. As the train moves with constant velocity, a frame attached to it will be inertial. With respect to train, the bob is in circular motion with necessary centripetal force provided by tension.

Use the formula, $\bar{V}_{b,G} = \bar{V}_{b,t} + \bar{V}_{t,G}$ to find velocity of bob with respect to ground.

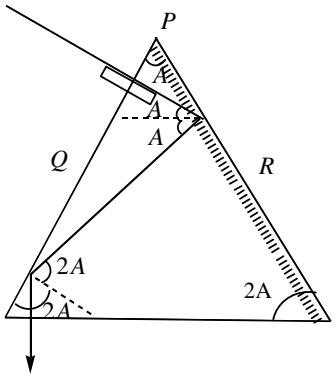


15. Intensity $I = \frac{nhf}{A}$ where n = no.of photons falling on the metal per second.

Saturation photo current $i \propto n$

$KE_{max} = hf - W_0$

Stopping potential $V_0 = \frac{h}{e}f - \frac{W_0}{e}$



$$\frac{\pi}{2} - \frac{A}{2} + \frac{\pi}{2} - 2A = \frac{\pi}{2}$$

$$\frac{5A}{2} = \frac{\pi}{2}$$

$$A = \frac{\pi}{5} = 36°$$

17. Let us denote the final volume and pressure by $V' = 1.25V_0$ and P', respectively. Taking helium to be an ideal gas, we can write that $P_0V_0 = P'V'$ under isothermal conditions, so that $P' = 0.8P_0$. Since the external pressure is $0.5P_0$, then net outward pressure on the surface of the balloon is $0.3P_0$. Thus the net pressure of the balloon is $0.3P_0$. Thus the net pressure force on half of the balloon is $0.3P\pi R^2$, where R is the final radius of the balloon, given by $1.25V_0 = 4\pi R^3/3$. But this must just balance the elastic force $\sigma\Delta A$, where $\Delta A = 2\pi Rt$ is the cross-sectional area of the balloon material at its mid-plane. Here t is the thickness of the balloon material, obtained from $\rho = m/(4\pi R^2 t)$.

$$(0.3P_0)\pi R^2 = \sigma(2\pi Rt)$$

$$\sigma = \frac{0.3P_0R}{2t} = \frac{0.3P_0R}{2}\left(\frac{4\pi R^2\rho}{m}\right) = \frac{3P_0\rho}{20m}(4\pi R^3) = \frac{3}{20}\frac{P_0\rho}{m}\left[\frac{5}{4}\times 3V_0\right]$$

$$\sigma = \frac{9}{16}\frac{P_0V_0\rho}{m} = \frac{9}{16}\frac{P_0(8\times 1500)}{18} = \frac{750}{2}P_0$$

18. $f_A = \dfrac{340-10}{340-10+10}\times 85 = \dfrac{330}{4} Hz$

$f_B = \dfrac{340+10}{340+10-10}\times 85 = \dfrac{350}{4} Hz$

$f_{beat} = f_B - f_A = 5Hz$

19. The change in internal energy of the gas is

$$\Delta U = 1.5nR(\Delta T) = 1.5(1mol)\left(\frac{25}{3} J/mol-K\right)(2K) = 25J$$

The heat given to the gas = 42 J

The work done by the gas is

$\Delta W = \Delta Q - \Delta U = 42J - 25J = 17J$

If the distance moved by the piston is x, the work done

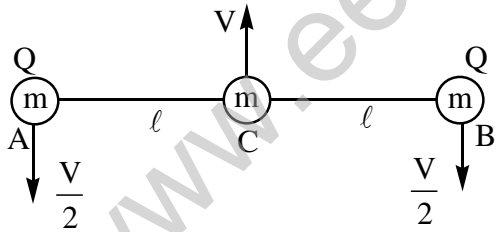
Is $\Delta W = (100kPa)(8.5cm^2)x$

Thus, $(10^5 N/m^2)(8.5\times 10^{-4} m^2)x = 17J$

Or $x = 0.2m = 20cm = 2\times 10^{-1} m$

20. Speed of 'C' is maximum when A,B and C become collinear with 'C' in between A and B. Initial momentum of system =0. Form conservation of linear momentum and symmetry A and B should move with half the speed of 'C' at that instant in opposite direction.

$KE_i + U_i = KE_f + U_f$



$$O + \frac{KQ^2}{\ell} = \frac{1}{2}mV^2 + \frac{1}{2}(2m)\left(\frac{V}{2}\right)^2 + \frac{KQ^2}{2\ell}$$

$$\Rightarrow \frac{3}{4}mV^2 = \frac{KQ^2}{\ell}\left[1-\frac{1}{2}\right]$$

$$\Rightarrow V^2 = \frac{2KQ^2}{3\ell m} = \frac{2\times 9\times 10^9\times 10^{-12}}{3\times\frac{3}{2}\times 10^{-3}} = 4 \Rightarrow V = 2ms^{-1}$$

CHEMISTRY

21. absence of H-bond

22. (A)

$$CH_3CH_2 \xrightarrow{Mg} CH_3CH_2MgBr \xrightarrow[2.H_3^+O]{1.H-\overset{\overset{O}{\|}}{C}-H} CH_3CH_2OH$$

23. (D)

It is example of EAS reactio0n which involves Arenium ion intermediate

24. (B)

Cyclisation occurs by attack of '-OH' group on carbonyl group. In structure b), the relative position of available 'OH' groups. Can form upto 5 membered ring

25. (A)

$$\left(P+\frac{aT^2}{V}\right)V^C = RT+b$$

$$P+\frac{aT^2}{V} = \frac{RT+b}{V^C}$$

$$P = (RT+b)V^{-C} - aT^2V^{-1}$$

So by comparing the equation with given equation

$A = RT+b$

$B = aT^2$

m = -C

n = -1

26. (A)

Let there be 1 mol of iron atom. We will have

mols of Fe(III) = (1/4) mol

mols of Fe(II) = (3/4) mol

Total positive charges = (l/4) (3) + (3/4) (2) = (9/4) mol

Let *a* be the amount of oxygen atoms. We have

Total negative charge = *a* (2)

To satisfy electrical neutrality, we have

Total positive charges = Total negative charge

$\frac{9}{4}$ mol = 2*a* or *a* = (9/8) mol

Hence, composition of the compound is $FeO_{9/8}$ or Fe_8O_9

27. (C)

$2A \rightleftharpoons B^+ + C^-$

$K = [B^+][C^-]$

$[B^+] = [C^-]$

$\Rightarrow [C]^2 = 10^{-30}$

$\Rightarrow [C^-] = 10^{-15}$

10^{-15} mole ions in 1000 cc

So 10^{-18} mole ions in 1cc

$\therefore\ 6.023\times10^{23}\times10^{-18}$ ions/cc

28. (C)

(P) In a group the tendency to exhibit inert pair effect increase downwards

(Q) PbI_4 : because of more polarazability of I^- it act as reducing agent.

(R) SiO_2 : PbO_2 and SnO_2 are amphoteric in nature

(S) NH_3 : In VA group hydrides the M-H bond energy decrease down words the group and thermal stability decreases.

29. (D) Conceptual

30. (A)

$Ag \rightarrow Ag^+ + e^-;\quad E^0 = -0.800V$

$AgI + e^- \rightarrow Ag + I^-;\ E^0 = -0.152V$

$AgI \rightarrow Ag^+ + I^-;\quad E^0 = -0.952V$

$$E = E^0 - \frac{0.0591}{n}\log Q$$

$$0 = -0.952 - \frac{0.0591}{1}\log K_{sp}$$

$$\log K_{sp} = -16.1$$

31. (C,D)

32. (A,B,C,D)

33. (B) (D)

34. (B,C)

35. (A,B,C)

$$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$$

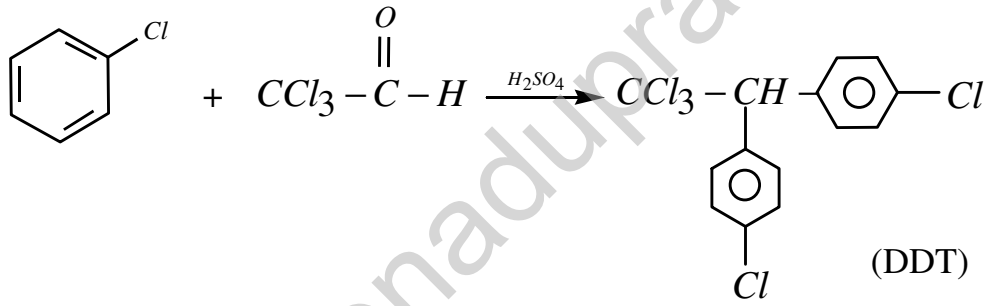
$$2NH_3 + 3Cuo \longrightarrow N_2 + 3H_2O + 3Cu$$

$$3Ba(N_3)_2 \xrightarrow{\Delta} Ba_3N_2 + 8N_2$$

36. $y = 5$

$$\Delta H = \frac{5}{2} \times R \times T = \frac{5}{2} \times 2 \times 300 = 1500\,\text{calorie} = 300y$$

37. D-Fructose α-L-Xylofuranose Maltose
L-Arabinose Lactose D-ribos

38 

(DDT)

39. the orbital that can satisfy all the given conditions is only one i.e. $3P_z$

40. (2)

$$B^{n+} \longrightarrow B^{(n+4)+}$$

Millimole at t = 0	a	0	$2e + B^{n+}$	$B^{(n-2)+}$
t = t	(a – x)	x	$5e + B^{(n+4)+}$	$B^{(n-1)+}$

Let normality be N for reducing agent.

Thus, at t = 0, $V_1N_1 = V_2N_2$

or, $a \times 2 = N \times 25$

$\therefore a = N$

and at t = t, $(a - x) \times 2 + x \times 5 = N \times 32$

$\therefore x = N$

Now, **$= 2.07 \times 10^{-2}\ \text{min}^{-1}$**

41. $t_n = \sum_{r=1}^{n} t_r - \sum_{r=1}^{n-1} t_r = \frac{n(n+1)(n+2)(n+3)}{8} - \frac{(n-1)n(n+1)(n+2)}{8} = \frac{n(n+1)(n+2)}{2}$

$\therefore \frac{1}{t_n} = \frac{2}{n(n+1)(n+2)}$ Define $a_n = \frac{2}{(n+1)(n+2)}$ then

$a_n - a_{n-1} = \frac{2}{(n+1)(n+2)} - \frac{2}{n(n+1)} = \frac{-4}{n(n+1)(n+2)}$

Thus $\frac{1}{t_n} = \frac{1}{2}(a_{n-1} - a_n)$. Hence $\sum_{r=1}^{n} \frac{1}{t_r} = \frac{1}{2}(a_0 - a_n)$

$= \frac{1}{2}\left(1 - \frac{2}{(n+1)(n+2)}\right) = \frac{n^2+3n}{2(n+1)(n+2)}$ (4)

42. $\sin\frac{2r\pi}{11} - i\cos\frac{2r\pi}{11} = -ie^{\frac{2r\pi}{11}i}$ Hence

$$\sum_{r=1}^{10}\left(\sin\frac{2r\pi}{11} - i\cos\frac{2r\pi}{11}\right) = -i\frac{e^{\frac{2\pi}{11}i}\left(\left(e^{\frac{2\pi i}{11}}\right)^{10} - 1\right)}{e^{\frac{2\pi i}{11}} - 1} = \frac{-i\left(1 - e^{\frac{2\pi i}{11}}\right)}{e^{\frac{2\pi i}{11}} - 1} = i$$

$$\therefore \sum_{p=1}^{32}(3p+2)\left\{\sum_{r=1}^{10}\left(\sin\frac{2r\pi}{11} - i\cos\frac{2r\pi}{11}\right)\right\}^p = \sum_{p=1}^{32}(3p+2)i^p$$

$= 3\sum_{p=1}^{32} pi^p + 2\sum_{p=1}^{32} i^p$

$= 3A + 2B \rightarrow (1)$

$A = i + 2i^2 + 3i^3 + \ldots\ldots + 32i^{32} = \frac{i(1-i^{32})}{(1-i)^2}\frac{32i}{-1-i} = \frac{-32i}{1-i} = 16(1-i)$

$B = \sum_{p=1}^{32} i^p = \frac{i\left(1-i^{32}\right)}{1-i} = 0$ *Ans:* $48(1-i)$

43. $\sum_{0\le i<j\le n}\sum(C_i+C_j)^2 = n\left(C_0^2+C_1^2+.....+C_n^2\right)+2\sum_{0\le i<j\le n}\sum C_iC_j = n\left(2n_{c_n}\right)+2\left(\frac{2^{2n}-2n_{c_n}}{2}\right)$

$=(n-1)2n_{c_n}+2^{2n}$

44. Total ways $=64_{c_2}$

Favourable ways $=2\{2(1+2+....+6)+7\}=98$

Hence probability $=\frac{98}{32\times63}=\frac{7}{144}$

45.

$$\left(\prod_{r=3}^{n}\frac{r-2}{r+2}\right)\left(\prod_{r=3}^{n}\frac{r^2+2r+4}{r^2-2r+4}\right)=\left(\frac{3-2}{3+2}\right)\left(\frac{4-2}{4+2}\right).....\left(\frac{n-2}{n+2}\right)\left(\frac{19}{7}\frac{28}{12}\frac{39}{19}.....\frac{n^2+2n+4}{n^2-2n+4}\right)$$

$$=\frac{1.2.3.4}{(n-1)n(n+1)(n+2)}.\frac{(n+3)\left(n^2+2n+1\right)}{7.12}$$

$$\therefore \lim_{n\to\infty}\prod_{r=3}^{n}\frac{r^3-8}{r^3+8}=\frac{2}{7}\lim_{n\to\infty}\frac{(n^2+3)\left(n^2+2n+4\right)}{(n-1)n(n+1)(n+2)}=\frac{2}{7}$$

46. Let $f(x)=\frac{x^2}{x^3+200}$ then $f^1(x)=\frac{x\left(400-x^3\right)}{\left(x^3+200\right)^2}$.

Hence $f^1(x)>0$ for $o<x<\sqrt[3]{400}$

And $f^1(x)<0\ for\ x>\sqrt[3]{400}$. Hence longest term is $a_7\ or\ a_8$

Since $a_7>a_8$ Maximum term $a_7=\frac{49}{543}$

47. The line $y=gx+\alpha$ passes through the centre $(-g,-g)$

$\Rightarrow -2g=-g^2+\alpha\Rightarrow\alpha=g^2-2g=(g-1)^2-1$

$\therefore \alpha\in[-1,\infty)$

48. Let $(x_i, y_i)(1 \le i \le 4)$ be the points of intersection of the given curves From the given equation $(x-1)^2 + \left(\frac{5}{x-1} + 4\right)^2 = r^2 \Rightarrow \sum_{i=1}^{4} x_i = 4$

Also $\frac{25}{(y-2)^2} + (y+2)^2 = r^2 \Rightarrow \sum_{i=1}^{4} y_i = 0$

$\therefore$ Centroid of ABC $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{4 - x_4}{3}, \frac{-y_4}{3}\right)$

This lies on the line $y = 3x - 4$

$\Rightarrow \frac{-y_4}{3} = (4 - x_4) - 4$ Hence locus of D is $y = 3x \rightarrow (1)$

49. $\overline{AB} = \hat{i} + 2\hat{j} + \hat{k}$ and $\overline{AC} = -6\hat{i} + 3\hat{j} - 3\hat{k}$

Then $AB = \sqrt{6}$ and $AC = \sqrt{54} = 3\sqrt{6}$ Now D divides BC in the ratio 1:3

Internally Hence $\overline{AD} = \frac{3\left(\hat{i} + 2\hat{j} + \hat{k}\right) + 1\left(-6\hat{i} + 3\hat{j} - 3\hat{k}\right)}{4} = \frac{-3\hat{i} + 9\hat{j}}{4}$

$\therefore AD = \frac{\sqrt{90}}{4} = \frac{3\sqrt{10}}{4}$ Ans : 4

50. $\cos^7 x + \cos^4 x - 2\cos^2 x = 0$

$\Rightarrow \cos^2 x = 0$ or $\cos^5 x + \cos^2 x - 2 = 0$

$\cos^2 x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{-\pi}{2}$

$\cos^5 x + \cos^2 x - 2 = 0$

$\Rightarrow \cos x = 1 \Rightarrow x = 2n\pi$

Hence the roots in $(-\pi, \pi)$ $-\frac{\pi}{2}, 0, \frac{\pi}{2}$

Ans :B

51. Put $1+x=\frac{1}{t}.Then$

$$I=\int\frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{1+\left(\frac{1-t}{t}\right)-\left(\frac{1-t}{t}\right)^2}}=\int\frac{-dt}{\sqrt{t^2+t-t^2-\left(1-2t+t^2\right)}}=-\int\frac{dt}{\sqrt{-1+3t-t^2}}$$

$$=-\int\frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{3}{2}\right)^2}}=-\sin^{-1}\left(\frac{2t-3}{\sqrt{5}}\right)+c=-\sin^{-1}\left(\frac{-3x-1}{\sqrt{5}\left(x+1\right)}\right)+c$$

$$=\sin^{-1}\left(\frac{3x+1}{\sqrt{5}\left(x+1\right)}\right)+c\Rightarrow\cos^{-1}\left(\frac{2\sqrt{1+x-x^2}}{\sqrt{5}\left(x+1\right)}\right)+c$$

Ans : A,D

51. Put $1+x=\frac{1}{t}.Then$

$$I=\int\frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{1+\left(\frac{1-t}{t}\right)-\left(\frac{1-t}{t}\right)^2}}=\int\frac{-dt}{\sqrt{t^2+t-t^2-\left(1-2t+t^2\right)}}=-\int\frac{dt}{\sqrt{-1+3t-t^2}}$$

$$=-\int\frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{3}{2}\right)^2}}=-\sin^{-1}\left(\frac{2t-3}{\sqrt{5}}\right)+c=-\sin^{-1}\left(\frac{-3x-1}{\sqrt{5}\left(x+1\right)}\right)+c$$

$$=\sin^{-1}\left(\frac{3x+1}{\sqrt{5}\left(x+1\right)}\right)+c\Rightarrow\cos^{-1}\left(\frac{2\sqrt{1+x-x^2}}{\sqrt{5}\left(x+1\right)}\right)+c$$

Ans : A,D

53. Let α,β be the roots of $x^2-ax-b=0$ Then $|\alpha|<1,|\beta|<1$ Now

$|b|=|\alpha\beta|<1$ also $f(-1)>0$ and $f(1)>0$

Thus $1+a-b>0\,and\,1-a-b>0\Rightarrow a+b<1\,and\,b-a<1$

Ans : A,B,C

54. Assume (i), That is $ABC=AC-CA$. Now

$ABC+A+B+C=AC-CA+\left(AB+BC+CA\right)=AC+AB+BC$

Thus $(A-I)(B-I)(C-I)=ABC-AB-BC-AC+A+B+C-I=-I$

$\therefore$ Inverse of $C-I=-(A-I)(B-I)\Rightarrow(C-I)(A-I)(B-I)=-I$

$CAB-CA-AB-CB+C+A+B-I=-I$

$CAB=CB-BC$

Then i and iii are equivalent. Similarly we get i and ii equivalent

Ans: ABCD

55. 1) $\pi+\tan^{-1}\left(\dfrac{x^4-x^2+1}{x\left(x^2-x^4-1\right)}\right)=\dfrac{3\pi}{4}\Rightarrow\tan^{-1}\left(\dfrac{-1}{x}\right)=\dfrac{-\pi}{4}\Rightarrow x=1$

2) $\displaystyle\sum_{k=1}^{n}\tan^{-1}\frac{1}{2k^2}=\sum_{k=1}^{n}\tan^{-1}\left(\frac{(2k+1)-(2k-1)}{1+(2k+1)(2k-1)}\right)=\sum_{k=1}^{n}\tan^{-1}(2k+1)-\tan^{-1}(2k-1)$

$\tan^{-1}(2n+1)-\tan^{-1}1=\tan^{-1}(2n+1)-\dfrac{\pi}{4}=\dfrac{\pi}{2}-\tan^{-1}\dfrac{1}{2n+1}-\dfrac{\pi}{4}$

$=\dfrac{\pi}{4}-\tan^{-1}\dfrac{1}{2n+1}$

3) $0\le x\le1\Rightarrow0\le\tan^{-1}x\le\dfrac{\pi}{4}\Rightarrow\dfrac{-\pi}{4}\le-\tan^{-1}x\le0$. Thus

$\dfrac{\pi}{2}-\dfrac{\pi}{4}\le\sin^{-1}x+\cos^{-1}x-\tan^{-1}x\le\dfrac{\pi}{2}-0\Rightarrow\dfrac{\pi}{4}\sum\sin^{-1}x+\cos^{-1}x-\tan^{-1}x\le\dfrac{\pi}{2}$

4) $\sin^{-1}\sin\dfrac{4\pi}{3}=\dfrac{-\pi}{3}$ and $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right)=\dfrac{5\pi}{6}$ Hence sum= $\dfrac{\pi}{2}$

58. $I=2\displaystyle\int_0^{\pi}\frac{2x\sin x}{1+\cos^2x}=4\int_0^{\pi}\frac{x\sin x}{1+\cos^2x}=4\pi\int_0^{\pi/2}\frac{\sin x}{1+\cos^2x}dx$

$=4\pi\left(-\tan^{-1}(\cos x)\right)_0^{\pi/2}=4\pi\left(0+\dfrac{\pi}{4}\right)=\pi^2\ \therefore[I]=9$

59. Tangent equation in $\dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}=1\rightarrow(1)$

Auxillallary circle is $x^2+y^2=a^2$

Homogonising we get $x^2+y^2-a^2\left(\dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}\right)^2=0$

These lines are perpendicular $\Rightarrow a^2\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)=2$

$\Rightarrow \cos^2\theta+\frac{a^2}{b^2}\sin^2\theta=2$

$\frac{a^2}{b^2}\sin^2\theta-\sin^2\theta=1\Rightarrow\frac{a^2}{b^2}=\frac{1+\sin^2\theta}{\sin^2\theta}$

$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\left(\frac{\sin^2\theta}{1+\sin^2\theta}\right)}=\sqrt{\frac{1}{1+\sin^2\theta}}$

$\therefore(\alpha+\beta)^2=4$

60. Given $\frac{\sin(A-B)}{Sin(A+B)}=\frac{\sin C-SinB}{SinC}\Rightarrow\sin(A-B)=\sin(A+B)-\sin B$

$\Rightarrow \sin B=2\cos A\sin B\Rightarrow\cos A=\frac{1}{2}\Rightarrow\tan A=\sqrt{3}$

$2Tan^2A=6$

* * *