SSC - CGL

QUANTITATIVE APTITUDE

SURDS

★ **Conjugate Surd:** If sum and product of two surds is a rational number then the two surds are called Conjugate surds to each other.

Ex: $5 + \sqrt{2}, \quad 5 - \sqrt{2}$

★ **Rationalizing factor:** If the product of two surds is a rational number then two surds are called Rationalizing factors to each other.

Ex: $\sqrt{7} + \sqrt{5}, \quad \sqrt{7} - \sqrt{5}$

★ **Note:** Every conjugate surd must be a rationalizing factor, but every rationalizing factor need not be a conjugate surd.

★ **I**f a is a positive rational number, 'n' is positive integer and $\sqrt[n]{a}$ is an irrational number. Then a $^{\frac{1}{n}}$ is called a surd of degree 'n'.

Ex: $\sqrt{7}$, (Quadratic surd) $\sqrt[3]{7}$, (Cubic surd) $\sqrt[4]{7}$ (Bi-quadratic surd)

Standard forms of a surd:

Square root:

$$\sqrt{a + 2.\sqrt{b}} = \sqrt{x} + \sqrt{y}$$

(where x + y = a, x . y = b)

$$\sqrt{a + 2.\sqrt{b} + 2.\sqrt{c} + 2.\sqrt{d}} = \sqrt{x} + \sqrt{y} + \sqrt{z}$$

(where x + y + z = a, x.y = b, y.z = c, z.x = d)

Cube root:

$$\sqrt[3]{a \pm b.\sqrt{c}} = x \pm \sqrt{y}$$

$$\left(\text{where } x = \sqrt{\frac{b-c}{3}} \text{ and } y = c \Leftrightarrow x^3 \pm 3xy = a\right)$$

Important formulae

1. $\sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a + \ldots\ldots \infty}}}} = \dfrac{\sqrt{1+4a} + 1}{2}$

2. $\sqrt{a - \sqrt{a - \sqrt{a - \sqrt{a - \ldots\ldots \infty}}}} = \dfrac{\sqrt{1+4a} - 1}{2}$

3. $\sqrt{a.\sqrt{a.\sqrt{a.\sqrt{a.\ldots\ldots \infty}}}} = a$

4. $\sqrt{a.\sqrt{a.\sqrt{a.\sqrt{a.\ldots \text{n times}}}}} = a^{\left(\frac{2^n - 1}{2^n}\right)}$

Surd	Rationalizing factor
$a + \sqrt{b}$	$a - \sqrt{b}$
$\sqrt[3]{a} + \sqrt[3]{b}$	$\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}$
$\sqrt[3]{a} - \sqrt[3]{b}$	$\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}$
$\sqrt[4]{a} + \sqrt[4]{b}$	$(\sqrt[4]{a} - \sqrt[4]{b})(\sqrt{a} + \sqrt{b})$
$\sqrt[4]{a} - \sqrt[4]{b}$	$(\sqrt[4]{a} + \sqrt[4]{b})(\sqrt{a} + \sqrt{b})$

QUESTIONS

1. Square root of $14 - 8\sqrt{3}$ is ....

a) $2\sqrt{2} + \sqrt{6}$ b) $2\sqrt{2} - \sqrt{6}$ c) $2\sqrt{3} + 6$ d) $8 + \sqrt{6}$

2. $\sqrt{12 + 2.\sqrt{15} + 4.\sqrt{5} + 4.\sqrt{3}}$

a) $\sqrt{3} + \sqrt{5} + \sqrt{15}$ b) $\sqrt{2} + \sqrt{3} + \sqrt{5}$ c) $2 + \sqrt{3} + \sqrt{5}$ d) $\sqrt{2} - \sqrt{3} - \sqrt{5}$

3. $\sqrt[3]{72 - 32.\sqrt{5}} = ?$

a) $3 - \sqrt{5}$ b) $6 - \sqrt{5}$ c) $5 - \sqrt{6}$ d) $3 - \sqrt{6}$

4. $\sqrt[6]{2\sqrt{2} + 3} \cdot \sqrt[3]{1 - \sqrt{2}}$

a) 1 b) $\sqrt{2}$ c) -1 d) $-\sqrt{2}$

5. $\sqrt{5.\sqrt{5.\sqrt{5.\sqrt{5.....\infty}}}} = ?$

a) 5 b) 1 c) $\sqrt{5}$ d) Can't estimate

6. $\sqrt{7.\sqrt{7.\sqrt{7.....10 \text{ times}}}} = a$

a) 7 b) $10.\sqrt{7}$ c) $7^{\left(1 - \frac{1}{2^{10}}\right)}$ d) $7^{\left(3 - \frac{1}{2^{10}}\right)}$

7. $\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20.....\infty}}}} = ?$

a) 3 b) 4 c) 5 d) 6

8. The conjugate surd of $\sqrt{2} - 1$ is

a) $-\sqrt{2} - 1$ b) $\sqrt{2} - 1$ c) $1 + \sqrt{3}$ d) $2 + \sqrt{3}$

9. The greatest among the following is ....

$\sqrt{22} + \sqrt{21}$, $\sqrt{23} + \sqrt{20}$, $\sqrt{24} + \sqrt{19}$

a) $\sqrt{22} + \sqrt{21}$ b) $\sqrt{23} + \sqrt{20}$ c) $\sqrt{24} + \sqrt{19}$ d) Can't estimate

10. The greatest among $\sqrt{21} - \sqrt{13}$, $\sqrt{19} - \sqrt{11}$, $\sqrt{13} - \sqrt{5}$, $\sqrt{23} - \sqrt{15}$

a) $\sqrt{21} - \sqrt{13}$ b) $\sqrt{19} - \sqrt{11}$ c) $\sqrt{13} - \sqrt{5}$ d) $\sqrt{23} - \sqrt{15}$

11. Arrange in decending order

$\sqrt[3]{4}$, $\sqrt{2}$, $\sqrt[6]{3}$, $\sqrt[4]{5}$

a) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$
b) $\sqrt[4]{5} > \sqrt[3]{4} > \sqrt[6]{3} > \sqrt{2}$
c) $\sqrt{2} > \sqrt[6]{3} > \sqrt[3]{4} > \sqrt[4]{5}$
d) $\sqrt[6]{3} > \sqrt[3]{4} > \sqrt{2} > \sqrt[4]{5}$

12. The geometric mean of $(\sqrt{100} + \sqrt{104})$, $(\sqrt{26} - 5)$ is

a) $\sqrt{2}$ b) $2.\sqrt[4]{6}$ c) $2.\sqrt{12}$ d) $2.\sqrt[4]{12}$

13. If $x = 4 - \sqrt{15}$ then $x^2 - 8x + 15 = ?$

a) 10 b) 11 c) 14 d) 16

14. If $x = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$ and $y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ then $x + y = ?$

a) 8 b) 16 c) $2.\sqrt{15}$ d) $2(\sqrt{5} + \sqrt{3})$

15. $\left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}\right)$ simplify

a) $2.\sqrt{6}$ b) $4.\sqrt{16}$ c) $2.\sqrt{3}$ d) $3.\sqrt{2}$

16. If $x = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ then $x^4 + \frac{1}{x^4} = ?$

a) 8642 b) 9602 c) 10104 d) 11132

17. Rationalizing factor of $\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}} + 1\right)$

a) $2^{\frac{2}{3}} + 2^{\frac{-2}{3}} + 1$ b) $2^{\frac{2}{3}} - 2^{\frac{-2}{3}} - 1$ c) $2^{\frac{1}{3}} + 2^{\frac{-1}{3}}$ d) $2^{\frac{1}{3}} + 1$

18. $\sqrt[3]{9.\sqrt{3} + 11.\sqrt{2}} = ?$

a) $3 + \sqrt{2}$ b) $\sqrt{3} + \sqrt{2}$ c) $2 + \sqrt{3}$ d) $11 + \sqrt{2}$

19. $\sqrt[6]{2} \div \sqrt[4]{6} = ?$

a) 1 b) $\sqrt[12]{\frac{2}{27}}$ c) $\sqrt[12]{\frac{27}{2}}$ d) $\sqrt[6]{\frac{2}{9}}$

20. $\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \ldots\ldots$ 99 terms.

a) 9 b) 10 c) $1 + \sqrt{99}$ d) $\sqrt{99} - 1$

21. If $x = 3 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$ then $x^3 - 9x^2 + 18x$ is

a) 10 b) 12 c) 14 d) 8

22. If $(7 + 4.\sqrt{3})^{x^2 - 8} + (7 - 4.\sqrt{3})^{x^2 - 8} = 14$ then $x = ?$

a) $\pm 1, \pm 3$ b) $\pm 3, \pm 7$ c) $\pm 3, \pm\sqrt{7}$ d) $\pm\sqrt{3}, \pm\sqrt{7}$

23. If $x = \sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3+.....\infty}}}}$ then

a) $x^2 - x + 3 = 0$ b) $x^2 - x - 3 = 0$ c) $x^2 + x - 3 = 0$ d) $x^2 + x + 3 = 0$

24. $x = \sqrt{30-\sqrt{30-\sqrt{30-\sqrt{30-....\infty}}}} = ?$

a) 3 b) 4 c) 5 d) 6

25. Pure surd of $2 . \sqrt[3]{5}$ is

a) $\sqrt[3]{20}$ b) $\sqrt{20}$ c) $\sqrt{40}$ d) $\sqrt[3]{40}$

EXPLANATIONS

1. $14 - 8\sqrt{3} = 14 - 2\sqrt{3 \times 16} = 14 - 2\sqrt{6 \times 8} = (6 + 8) - 2\sqrt{6 \times 8} = 2\sqrt{2} - \sqrt{6}$

2. $\sqrt{(3 + 4 + 5) + 2.\sqrt{3.5} + 2.\sqrt{3.4} + 2.\sqrt{4.5}} = \sqrt{3} + \sqrt{4} + \sqrt{5}$

3. From the concept of standard form of cube root of a surd (as mentioned above)

$x = \sqrt{\dfrac{32 - 5}{3}} = 3 \quad y = 5 \Rightarrow x - \sqrt{y} = 3 - \sqrt{5}$

4. $\sqrt[3]{\sqrt[2]{\sqrt[2]{2} + 3}} . \sqrt[3]{1 - \sqrt{2}}$

$= \sqrt[3]{1 + \sqrt{2}} . \sqrt[3]{1 - \sqrt{2}}$

$= \left\{(1 + \sqrt{2})(1 - \sqrt{2})\right\}^{1/3} = (-1)$

5. From important formulae No.3, answer is 5

6. From Important formulae No.4 answer is $= 7^{\left(1 - \frac{1}{2^{10}}\right)}$

7. From Important formulae no.1, answer is 5.

Shortcut: If any number which can be written as a product of two consecutive numbers then, sum of its square roots will be the greatest number of the two consecutive numbers.

Here, $20 = 4 \times 5 \Rightarrow \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20. ....\infty}}}} = 5$

8. $-\sqrt{2}$ - 1 is a conjugate surd of $\sqrt{2}$ - 1

(from synopsis).

9. The sum of two numbers of the surds are equal.

Hence, calculate the difference.

The surd with greater difference is greater.

$\Rightarrow \sqrt{24} + \sqrt{19}$ is greater.

10. When the differences of the numbers of the surds given are equal then, the surd with smaller numbers is greater.

$\Rightarrow \sqrt{13} - \sqrt{5}$ is greater.

11. LCM of powers of surds given is 12.

$4^{\frac{1}{3} \times 12} \quad 2^{\frac{1}{2} \times 12} \quad 3^{\frac{1}{6} \times 12} \quad 5^{\frac{1}{4} \times 12}$

$4^4, 2^6, 3^2, 5^3$

In descending order, $4^4 > 5^3 > 2^6 > 3^2 \Rightarrow \sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

12. The geometric mean of a & b is $\sqrt{ab}$.

Therefore, $\sqrt{(\sqrt{100} + \sqrt{104})(\sqrt{26} - 5)} = \sqrt{2}$

13. $x - 4 = -\sqrt{15}$ then, squaring on both sides

$x^2 - 8x + 16 = 15 \Rightarrow x^2 - 8x + 15 = 14$

14. From the formulae,

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} + \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$$

$$= 2\left(\frac{a + b}{a - b}\right) \Rightarrow 2\left(\frac{5 + 3}{5 - 3}\right) = 8.$$

15. From the formulae,

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} - \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$$

$$\Rightarrow \frac{4\sqrt{ab}}{a - b} \Rightarrow \frac{4 . \sqrt{3 \times 2}}{3 - 2} = 4 . \sqrt{6}$$

16. $x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

Hence, $x^2 + \frac{1}{x^2} = (10)^2 - 2 = 98,$

$$x^4 + \frac{1}{x^4} = 98^2 - 2 = 9604 - 2 = 9602$$

17. The Rationalising factor of $3\sqrt{a} + 3\sqrt{b}$ is $\left(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}\right)$

18. From the formulae,

$$\sqrt[3]{(x + 3y) . \sqrt{x} \pm (3x + y) . \sqrt{y}} = \sqrt{x} \pm \sqrt{y}$$

$$\sqrt[3]{(3 + 3 \times 2)\sqrt{3} + (3 + 3 \times 2)\sqrt{2}} = \sqrt{3} \pm \sqrt{2}$$

19. $\dfrac{\sqrt[6]{2^2}}{\sqrt[4]{6}} = \dfrac{2^{\frac{1}{3}}}{2^{\frac{1}{4}} . 3^{\frac{1}{4}}} = \dfrac{2^{\frac{1}{12}}}{3^{\frac{3}{12}}} = \sqrt[12]{\dfrac{2}{27}}$

20. $\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + ... + \dfrac{1}{\sqrt{99}+\sqrt{100}}$

(rationalising each term)

$\dfrac{\sqrt{2}-1}{1} + \dfrac{\sqrt{3}-\sqrt{2}}{1} + \dfrac{\sqrt{4}-\sqrt{3}}{1} + ... + \dfrac{\sqrt{100}-\sqrt{99}}{1} = \sqrt{100} - 1 = 9$

21. $x - 3 = 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$

$x^3 - 27 - 9x (x - 3) = 3 + 9 + 9 (x - 3)$

$x^3 - 9x^2 + 27x - 27 = 9x - 15$

$x^3 - 9x^2 + 18x = 12$

22. $(7 + 4\sqrt{3})^{x^2 - 8} + (7 - 4\sqrt{3})^{x^2 - 8}$

$= \left(7 + 4\sqrt{3}\right)^1 + \left(7 - 4\sqrt{3}\right)^1$ (or)

$= \left(7 + 4\sqrt{3}\right)^{-1} + \left(7 - 4\sqrt{3}\right)^{-1}$

$x^2 - 8 = 1$ or $x^2 - 8 = -1$

$x^2 = 9$ $\quad x^2 = 7$

$\Rightarrow x = \pm 3$ or $\pm\sqrt{7}$

23. Squaring on both sides, $x^2 = 3 + x$

$$x^2 - x - 3 = 0$$

24. From Important formula No.2. answer is 5.

Shortcut: If any number which can be written as a product of two consecutive numbers then, difference of the suqare roots will be the smallest number of the two consecutive numbers.

Here, $30 = 5 \times 6 = \sqrt{30 - \sqrt{30 - ......\infty}} = 5$

25. $2 .\sqrt[3]{5} = \sqrt[3]{2^3 \cdot 5} = .\sqrt[3]{40}$

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