SSC - CGL

Quantitative Aptitude

THEORY OF INDICES

In an exponential form (a^b), a is called base, b is called index (or) power (or) exponent.

★ If the given bases are same then, their powers are equal.

e.g.: $x^a = x^b$ then a = b

★ If the given powers are same then, their bases are equal.

e.g.: $a^n = b^n$ then a = b

Laws of Indices

1) $a^m \times a^n = a^{m+n}$
2) $a^m \div a^n = a^{m-n}$
3) $(a^m)^n = a^{mn}$
4) $a^{-n} = \frac{1}{a^n}$
5) $a^{\frac{1}{n}} = \sqrt[n]{a}$
6) $a^0 = 1$

Important formulae

★ $(a^m)^n \neq a^{m^n}$

★ $a^m = b^m \Leftrightarrow a = b$

★ $a^m = a^n \Leftrightarrow m = n$

★ $a^m = b \Leftrightarrow a = b^{\frac{1}{m}}$

★ $\sqrt[m]{a} \cdot \sqrt[n]{b} = \sqrt[mn]{a^n \cdot b^m}$

PROBLEMS

1. $(27)^{8.2} \times 3^x = 27^{10}$

1) 1.8 2) 5.4 3) 5.8 4) 1.6

2. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$ then, the value of x is?

1) 1 2) 0 3) 2 4) 3

3. $x^{a(b-c)} \cdot x^{b(c-a)} \cdot x^{c(a-b)} = ?$

1) 0 2) 1 3) $x^{a^2+b^2+c^2}$ 4) $x^{ab+bc+ca}$

4. $\frac{2.3^{n+1} + 7.3^{n-1}}{3^{n+2} - 2\left(\frac{1}{3}\right)^{1-n}} = ?$

1) 1 2) 3 3) −1 4) 0

5. $a^{\frac{2}{3}}\left\{a^{\frac{1}{3}}\left[a^{\frac{1}{4}}\right]^4\right\}^{\frac{1}{4}} = ?$

1) a 2) $\frac{1}{a}$ 3) $\frac{a}{2}$ 4) $-a$

6. The maximum of 33^{33}, $3^{3^{3^3}}$, 333^3, 3^{333}

1) 33^{33} 2) $3^{3^{3^3}}$ 3) 333^3 4) 3^{333}

7. If $2^x = 3^y = 6^{-z}$ then, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = ?$

1) 3 2) 2 3) 1 4) 0

8. If a + b + c = 0 then, $x^{a^2 b^{-1} c^{-1}} . x^{b^2 a^{-1} c^{-1}} . x^{c^2 a^{-1} b^{-1}} =$ ....

1) 0 2) 1 3) x 4) x^3

9. If x = 0.9 then, find the value of $\left\{1-\left[1-\left(1-x^3\right)^{-1}\right]^{-1}\right\}^{-\frac{1}{3}}$ = ...

1) 0.9 2) 0.3 3) 0.6 4) 0.081

10. What is the value of $\frac{10^n - 4^n}{5^n - 2^n} = ?$

1) 2^{n+1} 2) 2^n 3) 2^{n-1} 4) 2

11. $\frac{17 \times (2m^2n)^3}{(mn^3)^2 \times (4m^2)^2}$ simplify, when m = 19, n = 17

1) $\frac{1}{578}$ 2) $\frac{1}{1378}$ 3) $\frac{1}{493}$ 4) $\frac{1}{689}$

12. Find the values of a and b if, $(x^a)^2 = (x^4)^{2b}$ and $(\sqrt{x})^a = \frac{x^3}{x^b}$

1) a = 3, b = 1 2) a = 4, b = 2 3) a = 4, b = 1 4) a = 5, b = 3

13. If $\left(\sqrt{x^b}\right)^a = x^a \times x^b$. Find the relation between a and b.

1) $b = \frac{2a}{a-2}$ 2) $a^2 b - 2b = 3a$ 3) $b^2 a + 3b = 2a$ 4) $a = \frac{2b}{b-2}$

14. If $\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^4}\right)}}$, find the value of 'a'.

1) $\frac{4}{7}$ 2) $\frac{5}{6}$ 3) $\frac{3}{8}$ 4) $\frac{8}{9}$

15. $\frac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \frac{1}{x^{(a-b)} + x^{(c-b)} + 1} + \frac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$

1) 0 2) 1 3) x^{a-b-c} 4) None

16. Given $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$ then, z is approximately equal to ....

1) 2.9 2) 3.5 3) 2.5 4) 2

17. Find the value of $\left(x^{b+c}\right)^{b-c}\left(x^{c+a}\right)^{c-a}\left(x^{a+b}\right)^{a-b}$

1) 1 2) 2 3) −1 4) 0

18. If $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^{x}$ then, x equal to .....

1) $\frac{4}{9}$ 2) $\frac{2}{3}$ 3) $\frac{9}{4}$ 4) $\frac{3}{2}$

19. If $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$ then, x is equql to ......

1) $\frac{25}{4}$ 2) 4 3) 9 4) 16

20. $a^3b = abc = 180$. a, b, c are positive integers then, the value of c is ......

1) 110 2) 1 3) 4 4) 25

KEY

1-2; 2-3; 3-2; 4-1; 5-1; 6-2; 7-4; 8-4; 9-1; 10-2; 11-1; 12-3; 13-4; 14-2; 15-2; 16-1; 17-1; 18-3; 19-2; 20-2.

EXPLANATIONS

1-2; $3^{3\times 8.2} \times 3^{x} = 3^{3\times 10}$

$24.6 + x = 30 \Rightarrow x = 5.4$

2-3; $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{3-x}$

Bases are equal $\Rightarrow$ Powers are equal.

$x - 1 = 3 - x \Rightarrow x = 2$

3-2; $x^{(ab - ac)} . x^{(bc - ba)} . x^{(ca - cb)}$

$= x^{(ab - ac + bc - ba + ca - cb)}$

$= x^0 = 1$

4-1; $\frac{2.3^{(n+1)} + 7.3^{(n-1)}}{3^{(n+2)} - 2.3^{(n-1)}} = \frac{3^{(n-1)}(2.3^2 + 7)}{3^{(n-1)}(27 - 2)}$

$= \frac{25}{25} = 1$

[By taking $3^{(n-1)}$ common from both numerator and denominator]

5-1; $a^{\frac{2}{3}}\left[a^{\frac{1}{3}}\left(a^1\right)\right]^{\frac{1}{4}}$

$= a^{\frac{2}{3}}\left[a^{\frac{1}{3}+1}\right]^{\frac{1}{4}}$

$= a^{\frac{2}{3}}\left(a^{\frac{4}{3}\times\frac{1}{4}}\right)$

$= a^{\frac{2}{3}+\frac{1}{3}} = a$

6-2; **1)** $33^{33} = \left(3^3 \text{ to } 3^4\right)^{33}$

$= 3^{99}$ to 3^{132}

$\left(\because 33 \text{ is in between } 3^3 \text{ to } 3^4\right)$

2) $3^{3^{3^{3}}} = 3^{327}$

3) $333^3 = \left(3^5 \text{ to } 3^6\right)^3 = 3^{15}$ to 3^{18}

$\left(\because 333 \text{ is in between } 3^5 \text{ to } 3^6\right)$

4) 3^{333}

As all the numbers are powers of 3, compare (99 – 132), (15 – 18), 3^{27}, 333.

3^{27} is the highest number.

$\Rightarrow 3^{3^{3^{3}}}$ is the highest.

Shortcut:

Ignore the same bases (here 3), then consider the powers. The highest no. in the powers will finally be the greatest number.

7-4; $2^{(x)} = 3^{(y)} = 6^{(-z)} = K$

$\Rightarrow 2 = K^{\left(\frac{1}{x}\right)}, 3 = K^{\left(\frac{1}{y}\right)}, 2.3 = K^{-\left(\frac{1}{z}\right)}$

$2.3 = 6 \Rightarrow K^{\left(\frac{1}{x}\right)} . K^{\left(\frac{1}{y}\right)} = K^{\left(-\frac{1}{z}\right)}$

$\frac{1}{x} + \frac{1}{y} = -\frac{1}{z} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

8-4; $x^{\left(\frac{a^2}{bc}\right)} . x^{\left(\frac{b^2}{ac}\right)} . x^{\left(\frac{c^2}{ab}\right)}$

$= x^{\left(\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab}\right)}$

$= x^{\frac{a^3 + b^3 + c^3}{abc}} = x^{\frac{3abc}{abc}} = x^3$

$\left[\because a + b + c = 0 \Leftrightarrow a^3 + b^3 + c^3 = 3abc\right]$

Shortcut:

Given condition a + b + c = 0. Consider the values of a, b and c such that the condition is satisfied i.e., (a, b, c) can be (2, –1, –1) or (0, –1, +1), etc.....

Let us take a = b = c = 0

$x^{\left(\frac{a^2}{bc}\right)} . x^{\left(\frac{b^2}{ac}\right)} . x^{\left(\frac{c^2}{ab}\right)}$

$= x^{0 + 0 + 0} = 1$

9-1; $\left\{1 - \left[1 - \left(1 - x^3\right)^{-1}\right]^{-1}\right\}^{-\frac{1}{3}}$

$= \left\{1 - \left[1 - \left(\frac{1}{1 - x^3}\right)\right]^{-1}\right\}^{-\frac{1}{3}}$

$= \left\{1 - \left[\frac{-x^3}{1 - x^3}\right]^{-1}\right\}^{-\frac{1}{3}}$

$= \left\{1 + \frac{(1 - x^3)}{x^3}\right\}^{-\frac{1}{3}}$

$= \left[\frac{x^3 + 1 - x^3}{x^3}\right]^{-\frac{1}{3}}$

$= x^{-3 \times \left(-\frac{1}{3}\right)} = x$, i.e,. x = 0.9. Hence, 0.9.

10-2; Taking 2^n common in the numerator.

$$\frac{2^n (5^n - 2^n)}{(5^n - 2^n)} = 2^n$$

11-1; $\frac{17 \times 8 (m)^6 \times (n)^3}{(m^2) \times (n^6) \times 16 \times (m^4)} = \frac{17 \times m^6 \times n^3}{2 \times m^6 \times n^6}$

$$= \frac{17}{2n^3}$$

substituting n = 17 then, $\frac{1}{2 (17)^2} = \frac{1}{578}$

12-3; $x^{(2a)} = x^{(8b)} \Rightarrow a = 4b$

$x^{\left(\frac{a}{2}\right)} = x^{(3 - b)} \Rightarrow a = 6 - 2b$

$4b = 6 - 2b \Rightarrow b = 1, a = 4$

13-4; $x^{\left(\frac{ab}{2}\right)} = x^{(a + b)}$

$\Rightarrow \frac{ab}{2} = a + b \Rightarrow a (b - 2) = 2b$

$\Rightarrow a = \frac{2b}{b - 2}$

14-2; $x^{\left(5 + a - \frac{1}{2}\right)} = \left(\frac{x^{(8a)}}{x^{(-4)}}\right)^{\frac{1}{2}}$

$\Rightarrow x^{\left(5 + a - \frac{1}{2}\right)} = x^{(4a + 2)}$

$\frac{9}{2} + a = 4a + 2 \Rightarrow a = \frac{5}{6}$

15-2; $\frac{1}{\left(1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}\right)} + \frac{1}{\left(1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}\right)} + \frac{1}{\left(1 + \frac{x^a}{x^c} + \frac{x^b}{x^c}\right)}$

$= \frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^a + x^b + x^c} + \frac{x^c}{x^a + x^b + x^c}$

$= \frac{x^a + x^b + x^c}{x^a + x^b + x^c} = 1$

(or)

Assuming the values of a = b = c = 0

Hence, solving

$$\frac{1}{\left(1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}\right)} + \frac{1}{\left(1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}\right)} + \frac{1}{\left(1 + \frac{x^a}{x^c} + \frac{x^b}{x^c}\right)} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

16-1; $x^z = y^2 \Leftrightarrow 10^{(0.48z)} = 10^{(2 \times 0.70)}$

$\Rightarrow 0.48\,z = 1.40 \Rightarrow z = \frac{1.40}{0.48} = 2.9$ (approx)

17-1; $x^{(b^2 - c^2)} . x^{(c^2 - a^2)} . x^{(a^2 - b^2)}$

$\Rightarrow x^0 = 1$

(or) Let a = b = c = 0

$x^0 . x^0 . x^0 = 1$

18-3; $x^{(x^{3/2})} = (x^{3/2})^x \Rightarrow x^{(3/2)} = \frac{3}{2}(x)$

$\Rightarrow x^{(1/2)} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$

19-2; $5^2 + 12^2 = 13^2$

(from the pythogorous triplet)

Hence, $\sqrt{x} = 2 \Rightarrow x = 4$

20-2; $a^3.b = a . b . c \Rightarrow a^2 = c$ (where a, c are integers)

To satisfy this condition a, c values must be 1.

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